Welfare-based optimal monetary policy with unemployment and sticky prices: A linear-quadratic framework

Federico Ravenna and Carl E. Walsh*

This draft: 11 May 2010

Abstract

We derive a linear-quadratic model that is consistent with sticky prices and search and matching frictions in the labor market. We show that the second-order approximation to the welfare of the representative agent depends on inflation and "gaps" that involve current and lagged unemployment. Our approximation makes explicit how welfare costs are generated by the presence of search frictions. These costs are distinct from the costs associated with relative price dispersion and fluctuations in consumption that appear in standard new Keynesian models. We show the labor market structure has important implications for optimal monetary policy. (JEL: E52, E58, J64).

*Department of Economics, University of California, Santa Cruz, CA 95064; fravenna@ucsc.edu, walshc@ucsc.edu.
We thank seminar participants at UC Irvine, LSU, and Richard Dennis, Bart Hobijn and Michael Woodford for helpful comments and the Banque de France Foundation for financial support.

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The steep increases in unemployment associated with the financial crisis and global recession of 2008-2009, and the wide-spread focus on unemployment in both the popular press and in policy debates, is in sharp contrast to the canonical new Keynesian model in which unemployment is noticeably absent. In that model, workers are never unemployed and only hours worked per worker vary over the business cycle. As a consequence, the basic new Keynesian (NK) model cannot shed light on whether monetary policy should respond to the unemployment rate or whether there is a role for stabilizing unemployment fluctuations that is distinct from stabilizing fluctuations in inflation and the consumption gap as in standard NK models.

Our objective in this paper is to explore the implications for monetary policy of a model with sticky prices and search-based unemployment. We first show how such a model can be reduced to a linear expectational-IS curve and a Phillips curve linking inflation and the gaps between unemployment, expected future unemployment, and lagged unemployment relative to their efficient levels. The coefficients in these two relationships depend on the underlying structural parameters of the model that govern preferences, the degree of nominal price rigidity, and the search and bargaining processes in the labor market.

We then derive a second-order approximation to the welfare of the representative household and show that, in addition to the standard inflation and consumption gap terms, a new term appears that involves labor market tightness. This new term captures all the welfare costs associated with labor market search inefficiency. In a standard new Keynesian model, inflation leads to an inefficient composition of market consumption because of the dispersion of relative prices that inflation causes. Search frictions generate an inefficient composition of aggregate utility because an alternative to market consumption (home production in our specification) is available to unemployed agents, and this alternative does not suffer from the search friction necessary to produce employment matches and market consumption. This inefficiency is distinct from the inefficient composition of market consumption generated by inflation and so results in an additional objective in the loss function. The first best is attained when both inflation and the gap between unemployment and its efficient level are always equal to zero. However, because labor market frictions introduce a new state variable, optimal policy involves smoothing a quasi-difference in the level of this unemployment gap. Thus, neither the level of unemployment nor simply the level of the unemployment gap correctly measures the appropriate objective of monetary policy.

In a standard NK model, fluctuations in employment, consumption, and output all move in proportion to one another relative to their flexible-price counterparts. Thus, fluctuations in welfare could be equivalently expressed in terms any one of these variables, together with inflation volatility. With search frictions, this equivalence does not hold, so the unemployment gap term we obtain in the welfare approximation cannot be replaced with a consumption gap term. Each gap plays a distinct role in affecting welfare, and to properly evaluate this distinction, one needs to be able to assess the relative importance of consumption gap stabilization versus unemployment gap stabilization. By developing a quadratic approximation to welfare, we obtain explicit expressions for the relative weight on each and can assess how this weight varies with structural characteristics of the labor market. Beside affecting the goals of the policy maker, search frictions also change the monetary transmission mechanism by adding a cost channel for the interest rate along with the traditional demand channel.[1]

Given the linear representation of the structural equations and a model-consistent quadratic loss function, the framework can be used to study monetary policy issues in the same way the standard NK model has been used. In light of the empirical evidence from DSGE models with labor market search frictions for the U.S. (Sala, Söderström, and Trigari 2008) and for the Euro area (Christoffel, Kuester, and Linzert 2009), we allow for stochastic fluctuations in the relative bargaining power of workers and firms. This shock distorts the flexible-price equilibrium, and thus introduces a third distortion (the other two being monopolistic competition and sticky prices) that is absent from standard NK models. Shocks to bargaining power generate policy trade offs in our model, just as cost shocks do in standard NK models.

In a basic NK model, cost-push shocks can lead to large losses if the central bank pursues a single-minded focus on price stability. We find, however, that if cost-push shocks reflect random fluctuations in the relative bargaining power of workers and firms, price stability is nearly optimal. The reason is closely related to the argument made by Goodfriend and King (2001) that the long-term nature of employment relationships reduces the welfare costs of temporary deviations of the contemporaneous marginal product of labor and the marginal rate of substitution between leisure and consumption. With efficient bargaining but fluctuations in bargaining shares, price stability remains close to the optimal policy.

We find that a policy designed to minimize volatility in inflation and in inefficient fluctuations of unemployment — policy objectives used in some of the existing literature — targets the wrong measure of search inefficiency and can produce a significant reduction in welfare.[2] And in contrast to the results obtained in the staggered price and wage adjustment model of Erceg, Henderson and Levin (2000), a simple Taylor rule results in a welfare loss that is much higher than that achieved under the optimal policy. In fact, the backward-looking policy rule estimated by Clarida, Galí and Gertler (2000) for the Volcker-Greenspan era generates welfare loss equal to nearly 2 percent of steady-state consumption compared to essentially no loss under a policy of price stability.

A growing number of papers have incorporated unemployment into NK models. Examples include Chéron and Langot (1999), Walsh (2003, 2005), Christoffel, Kuester, and Linzert (2006), Blanchard and Galí (2006), Krause and Lubik (2007), Faia (2007), Krause, Lubik, and Lopéz-Salido (2007), Ravenna and Walsh (2008), Sala, Söderström, and Trigari (2008), Thomas (2008), Gertler,

[1] A cost channel arises when firms' marginal cost depends directly on the interest rate as, for example, in Christiano, Eichenbaum, and Evans (2005). The policy implications of a cost channel in a model without labor market frictions are discussed in Ravenna and Walsh (2006).

[2] While we focus on optimal policy, the presence of labor market search frictions also affects some of the standard properties of simple Taylor rules. For example, the conditions for determinacy do not generally satisfy the so-called Taylor principle. See Kurozumi and Van Zandweghe (2008) for an analysis of determinacy in model that is quite similar in structure to the model we develop here.

Sala, and Trigari (2008), Gertler and Trigari (2009), and Trigari (2009). The focus of these earlier contributions has extended from exploring the implications for macro dynamics in calibrated models to the estimation of DSGE models with labor market frictions. For example, Sala, Söderström, and Trigari (2008) evaluate monetary policy trade-offs and optimal policy in an estimated model with search and matching frictions in the labor market, but they use an ad hoc quadratic loss function rather than the model consistent welfare approximation we derive.

The papers closest in motivation to ours are Blanchard and Galí (2010) and Thomas (2008). Both these papers make specific assumptions on how the wage setting process generates inefficient fluctuations in the way the surplus from an employment match is shared between the worker and the firm. Our approach does not take a stand on the sources of these fluctuations, and instead assumes they are exogenous, a strategy already pursued by Shimer (2005). Many authors, including Blanchard and Galí (2010) and Thomas (2008), have assumed these fluctuations reflect some form of real wage rigidity, but the role of wage stickiness in accounting for macroeconomic fluctuations is a topic of active debate. Shimer (2005) demonstrated that matching models with wages set by Nash bargaining cannot generate the level of unemployment volatility seen in the data, and imposing wage rigidity increases the volatility of unemployment. However, Pissarides (2009) concludes that wage stickiness does not explain the unemployment volatility puzzle. To highlight the implications of search frictions in a model that is otherwise well known, we follow the standard NK model and do not impose constraints on wage adjustment. Instead, we assume stochastic fluctuations in worker-firm bargaining shares. These fluctuations can also be interpreted as representing deviations of the real wage from its efficient level and so capture some of the same effects generated by assuming real wage rigidity.

There are other differences between the model we employ and those developed by Blanchard and Galí (henceforth BG) and Thomas. BG share with our paper the goal of developing a simple framework akin to the basic NK model but in which unemployment plays a central role. In contrast to the Mortensen-Pissarides search model we employ, BG assume firms face hiring costs that are increasing in the degree of labor market tightness (measured as new hires relative to unemployment). BG assume offsetting income and substitution effects on labor supply, implying unemployment remains constant in the face of productivity shocks when prices are flexible. This implies that monetary policy should focus on stabilizing the *level* of unemployment, as well as inflation. Our model allows unemployment to fluctuate under flexible prices, but because productivity causes the efficient level of unemployment to fluctuate, the appropriate objective of policy is defined in terms of an unemployment rate *gap* that is more comparable to the output gap appearing in standard NK models. We also find that both the current unemployment gap and its lagged value are relevant for welfare; because of search frictions, the number of unemployed workers at the end of the previous period is an endogenous state variable.

In addition, the search and matching framework is, in our view, better able to link labor market characteristics to macroeconomic behavior than the hiring costs approach used by BG. For example, the roles of vacancies, job turnover, unemployment benefits, and job-finding probabilities are explicit in our model. The welfare approximation in BG also relies on the assumption that hiring costs are of second order magnitude, an assumption we can dispense with.

Thomas (2008) incorporates convex costs of posting vacancies and staggered real wage adjustment and derives a quadratic welfare approximation in terms of squared deviations of variables from their steady-state values. In contrast, our approach, besides yielding an expression for the welfare loss that is simpler in form, shows explicitly how each variable appearing in the objective function can be expressed in terms of a squared deviation from its *efficient* level. This helps to highlight that policy involves stabilizing real variables around time-varying efficient levels, not constant steady-state levels, and that optimal policy involves closing gaps.

Two further issues merit brief discussion before beginning our analysis. First, all the existing literature that incorporates unemployment into models with nominal rigidities has assumed households are able to insure against idiosyncratic consumption risk. Thus, an agent's consumption is independent of employment status. We too follow the literature in making this assumption. A full understanding of the welfare costs of unemployment will undoubtedly require a recognition of heterogeneity and imperfect consumption insurance. Doing so is beyond the scope of the present paper, but it is clearly an important topic for future research.

Second, even in the context of a model that deviates from the basic NK model along one dimension, the derivation of a second-order approximation to the welfare of the representative agent expressed in terms of efficiency gaps becomes quite complex. In our view, the benefits of such a derivation outweigh the costs as the linear-quadratic approach has proven immensely useful in providing insights relevant for monetary policy design. For example, the approach has helped highlight the role of distortions in affecting the relative weight placed on inflation versus output volatility, clarified the definition of output around which actual output should be stabilized, and facilitated the analysis of optimal policy design.[3]

The rest of the paper is organized as follows. Section 1 presents the basic model, derives a log-linearized version of the model, and discusses the connections between labor market structure and the Phillip curve. The model-consistent welfare approximation and optimal policy are studied in section 2. The impact of labor market structure on optimal policy is investigated in section 3, while conclusions are summarized in section 4.

1 The model economy

The model consists of 1) households whose utility depends on the consumption of market and home produced goods; 2) firms who employ labor to produce a wholesale good which is sold in a competitive market; and 3) retail firms who transform the wholesale good into differentiated final goods sold to households in an environment of monopolistic competition. The labor market is characterized by search frictions. Households members are either employed (in a match) or

[3] Woodford (2003) develops the linear-quadratic approach and illustrates its value for policy analysis. For further examples of the approach's value, see chapter 8 of Walsh (2010).

searching for a new match. Retail firms adjust prices according to a standard Calvo specification. The modelling strategy of locating labor market frictions in the wholesale sector where prices are flexible and locating sticky prices in the retail sector among firms who do not employ labor provides a convenient separation of the two frictions in the model. A similar approach was adopted in Walsh (2003, 2005), Ravenna and Walsh (2008), Thomas (2008), and Trigari (2009).

1.1 Final goods

The demand for final goods arises from two sources – households who purchase retail goods to form a consumption bundle and wholesale firms who must employ real resources to recruit and hire workers.

Households Households consist of a large number of members who can be either employed by wholesale firms in production activities or unemployed. In the former case, they receive a market real wage w_t; in the latter case, they receive a fixed amount w^u of household production units. As is standard in the literature on matching frictions, we assume that consumption risks are fully pooled. The household's instantaneous utility at time t is given by the preference specification

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma},$$

where total consumption C_t depends on consumption of market goods C_t^m and home production $w^u(1-N_t)$:

$$C_t = C_t^m + w^u(1-N_t), \tag{1}$$

where N_t is the number of household members employed during the period. Market consumption is an aggregate of goods purchased from the continuum of retail firms, indexed by j, that produce differentiated final goods:

$$C_t^m \leq \left[\int_0^1 C_t^m(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Intratemporal optimal choice across goods implies

$$C_t^m(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} C_t^m, \tag{2}$$

where

$$P_t \equiv \left[\int_0^1 P_t(j)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

Households maximize expected discounted utility, and the intertemporal first order condition for the households' decision problem yield the standard Euler equations:

$$\lambda_t = \beta \mathrm{E}_t\{R_t \lambda_{t+1}\} \tag{3}$$
$$\lambda_t = \beta \mathrm{E}_t\left\{i_t \frac{P_t}{P_{t+1}} \lambda_{t+1}\right\}, \tag{4}$$

where R_t (i_t) is the gross return on an asset paying one unit of the consumption aggregate (currency) in any state of the world and λ_t is the marginal utility of consumption.

Wholesale firms Firms in the wholesale sector produce output using labor through the production function $Y_t^w = Z_t N_t$, where Z_t is an exogenous stationary productivity shock common to all firms. The production process also requires firms to pay a per-period cost to post employment vacancies. We assume job postings are homogenous with the final goods produced by the retail sector. To post v_t vacancies, wholesale firms buy individual final goods $v_t(j)$ from each j final-goods-producing retail firm subject to the constraint

$$\left[\int_0^1 v_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \geq v_t. \tag{5}$$

Total expenditure on job posting costs is given by

$$\kappa \int_0^1 P_t(j) v_t(j) dj$$

which wholesale firms minimize subject to (5) for any choice of v_t. The demand by wholesale firms for the final goods produced by retail firm j is given by

$$v_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} v_t. \tag{6}$$

and at the optimum the cost to keeping a vacancy open in period t can be written as κP_t.

Total expenditure on final goods by households and wholesale firms is

$$\int_0^1 P_t(j) C_t^m(j) dj + \kappa \int_0^1 P_t(j) v_t(j) dj = \int_0^1 P_t(j) Y_t^d(j) dj = P_t(C_t^m + \kappa v_t)$$

where $Y_t^d(j) \equiv C_t^m(j) + \kappa v_t(j)$ is total demand for final good j.

Retail firms Retail firms purchase wholesale output at price P_t^w in a competitive market. The wholesale good is then converted into a differentiated final good that is sold to households and wholesale firms. Retail firms maximize profits subject to a CRS technology for converting wholesale goods into final goods, the demand functions (2) and (6), and a restriction on the frequency with which they can adjust their price.

Each period a firm can adjust its price with probability $1 - \omega$. A retail firm that can adjust its price in period t chooses $P_t(j)$ to maximize

$$\sum_{i=0}^{\infty} (\omega\beta)^i \, \mathrm{E}_t \left[\left(\frac{\lambda_{t+i}}{\lambda_t} \right) \left(\frac{(1+\tau)P_t(j) - P^w_{t+i}}{P_{t+i}} \right) Y_{t+i}(j) \right]$$

subject to

$$Y_{t+i}(j) = Y^d_{t+i}(j) = \left[\frac{P_t(j)}{P_{t+i}} \right]^{-\varepsilon} Y^d_{t+i} \tag{7}$$

where Y_t^d is aggregate demand for the final goods basket and we assume the firm's output is subsidized at the fixed rate τ. This subsidy will be employed when we wish to ensure the steady-state equilibrium is efficient. The real marginal cost for retail firms is the price of the wholesale good relative to the price of final output, P_t^w/P_t. The standard pricing equation obtains which, when linearized around a zero-inflation steady state yields a NK Phillips curve in which the retail price markup

$$\mu_t \equiv \frac{P_t}{P_t^w}$$

is the driving force for inflation. As in a standard Phillips curve, the elasticity of inflation with respect to real marginal costs will be $\delta \equiv (1-\omega)(1-\beta\omega)/\omega$.

Market clearing Goods market clearing requires that household consumption of market produced goods plus final goods purchased by wholesale firms to cover the costs of posting job vacancies equal the output of the retail sector:

$$Y_t = C_t^m + \kappa v_t = C_t - w^u (1 - N_t) + \kappa v_t, \tag{8}$$

where v_t is the aggregate number of vacancies posted and κ is the cost per vacancy.

1.2 Wholesale goods, employment and wages

The labor market is characterized by search frictions. At the beginning of each period t a share ρ of the matches N_{t-1} that produced output in period $t-1$ breaks up. Workers not in a productive match at $t-1$ or who do not survive the exogenous separation hazard at the start of the period seek new matches.[4] Thus, the number of job seekers in period t is

$$u_t \equiv 1 - (1-\rho)\, N_{t-1}. \tag{9}$$

[4]By incorporating only a constant rate of exogenous separations, we follow most of the literature that has embedded labor search into monetary policy models. There is, of course, an active debate on the relative importance of endogenous fluctuations in unemployment inflows and outflows at business cycle frequencies; see Davis and Haltiwanger (1992), Shimer (2007), and Elsby, Michaels, and Solon (2009). For a monetary model with endogenous job destruction, see Walsh (2003, 2005).

Note that u_t is a predetermined variable as of time t.[5] Unemployed workers are matched stochastically with job vacancies. The matching process is represented by a CRS matching function

$$m(u_t, v_t) = \chi v_t^{\alpha} u_t^{1-\alpha} = \chi \theta_t^{\alpha} u_t \tag{10}$$

where $\theta_t \equiv v_t/u_t$ is the measure of labor market tightness, and $0 < \alpha < 1$. The number of matches that produce in period t is

$$N_t = (1-\rho)\, N_{t-1} + m(u_t, v_t). \tag{11}$$

To hire workers, wholesale firms must post vacancies. Given that the size of the firm is indeterminate with constant returns to scale, we can focus on the firm's decision to hire an additional worker. With free entry, the value of a vacancy is zero in equilibrium. This so-called job posting condition implies that the expected value of a filled job will equal the cost of posting a vacancy, or

$$q_t J_t = \kappa,$$

where J_t is the value of a filled job, $q_t \equiv m_t/v_t$ is the probability a firm with a vacancy will fill it, and κ is the cost of posting a vacancy. The value of a filled job is also equal to the firm's current period profit plus the discounted value of having a match in the following period. If a job produces output Z_t and w_t is the wage paid to the worker, than the value of a filled job in terms of final goods is

$$J_t = \left(\frac{P_t^w}{P_t}\right) Z_t - w_t + (1-\rho)\beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) J_{t+1}, \tag{12}$$

or

$$\frac{Z_t}{\mu_t} = w_t + \frac{\kappa}{q_t} - (1-\rho)\beta \mathrm{E}_t \left(\frac{1}{R_t}\right)\left(\frac{\kappa}{q_{t+1}}\right) \tag{13}$$

where $R_t^{-1} \equiv \beta\left(\lambda_{t+1}/\lambda_t\right)$ is the stochastic discount factor, and both wages and vacancies are measured in terms of the retail goods basket. The left side of (13) is the marginal product of a worker. The right side is the marginal cost of a worker to the firm. In the absence of labor market frictions, this cost would just be the real wage, and one would have $Z_t/\mu_t = w_t$, or $1/\mu_t = w_t/Z_t$; this corresponds to the standard NK model, where the real marginal cost variable that drives inflation is the real wage divided by labor productivity. With labor market frictions, additional factors come into play. According to (13), the cost of labor also includes the search costs associated with hiring (κ/q_t) and the discounted recruitment cost savings if an existing employment match survives into the following period.

The real wage appears in (13). A standard approach allowing for flexible wages is to assume Nash bargaining between firms and workers in which each participant receives a fixed share of the total surplus. In this case, Shimer (2005) pointed out that the real wage responds strongly to

[5] We take the number of job seekers u_t as our measure of unemployment. The standard measure of unemployment would more closely match the number of workers not in a match at the end of the period, $1 - N_t$. The two are related since u_{t+1} is equal to $1 - N_t$ plus the number of exogenous separations ρN_t.

productivity shocks, leaving unemployment much less volatile than in the data. In light of the "Shimer puzzle," many authors have introduced some form of real wage rigidity (see for example Hall, 2005, Gertler and Trigari, 2009). Since our objective is to develop a simple framework that parallels the basic NK model yet incorporates unemployment, we will follow the literature that assumes Nash bargaining over wages. This choice is consistent with the assumption of flexible wages underlying the basic NK model and allows a straightforward comparison of the policy implications of the two frameworks. We deviate from the standard assumption of fixed bargaining weights, however, by allowing the division of a match surplus to vary stochastically.[6]

The value of a match to a firm is J_t. For a worker, if $p_t \equiv m_t/u_t$ denotes the job finding probability of an unemployed worker, the valuation equation for being in a match that produces in period t is

$$V_t^E = w_t + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left\{ (1-\rho) V_{t+1}^E + \rho \left[p_{t+1} V_{t+1}^E + (1-p_{t+1}) V_{t+1}^U \right] \right\},$$

since a matched worker survives the exogenous separation hazard with probability $1-\rho$, is exogenously separated with probability ρ but finds another match with probability p_{t+1}, and fails to find a match with probability $1-p_{t+1}$. The valuation equation for being unmatched is

$$V_t^U = w^u + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left\{ (1-p_{t+1}) V_{t+1}^U + p_{t+1} \left[(1-\rho) V_{t+1}^E + \rho V_{t+1}^U \right] \right\},$$

since with probability $1-p_{t+1}$ the worker fails to find a match and with probability p_{t+1} a match is made and survives the exogenous separation hazard with probability $1-\rho$ and fails to with probability ρ. Thus, the surplus value of a match to a worker is

$$V_t^S \equiv V_t^E - V_t^U = w_t - w^u + \beta(1-\rho) \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) (1-p_{t+1}) V_{t+1}^S.$$

Let b_t denote the worker's share of the job surplus in period t, where b_t is assumed to follow a stationary stochastic process. Under Nash bargaining, the sharing rule implies

$$(1-b_t)\left(V_t^E - V_t^U\right) = b_t J_t = b_t \left(\frac{\kappa}{q_t} \right). \tag{14}$$

But noting that $p_{t+1} = \theta_{t+1} q_{t+1}$, the equilibrium real wage under Nash bargaining that satisfies (14) is[7]

$$w_t = w^u + \left(\frac{b_t}{1-b_t} \right) \left(\frac{\kappa}{q_t} \right) - (1-\rho) \left(\frac{1}{R_t} \right) \mathrm{E}_t \left(1-p_{t+1}\right) \left(\frac{b_{t+1}}{1-b_{t+1}} \right) \left(\frac{\kappa}{q_{t+1}} \right). \tag{15}$$

[6] Christoffel, Kuester, and Linzert (2009) show that in an estimated DSGE model of the Euro area with search friction, bargaining shocks play a significant role in output and inflation fluctuations, both in absolute terms and relative to other labor market disturbances. In our model, where wages are Nash-bargained in every period, bargaining shocks increase the volatility of employment relative to output.

[7] Details are provided in the appendix available from the authors.

Substituting (15) into (13), one finds that the relative price of wholesale goods in terms of retail goods is equal to

$$\frac{P_t^w}{P_t} = \frac{1}{\mu_t} = \frac{\xi_t}{Z_t}, \tag{16}$$

where ξ_t is the effective cost of labor and is defined as

$$\xi_t \equiv w^u + \left(\frac{1}{1-b_t}\right)\left(\frac{\kappa}{q_t}\right) - (1-\rho)\left(\frac{1}{R_t}\right)\mathrm{E}_t\left(\frac{1-b_{t+1}p_{t+1}}{1-b_{t+1}}\right)\left(\frac{\kappa}{q_{t+1}}\right). \tag{17}$$

Labor market tightness affects inflation through ξ_t. A rise in labor market tightness reduces q_t, the probability a firm fills a vacancy, and raises the value of a filled job (κ/q_t). This increases wages in the wholesale sector and raises wholesale prices relative to retail prices. The resulting rise in the marginal cost of the retail firms and fall in the retail price markup increases inflation. Expectations of greater labor market tightness in the future increase the expected cost of hiring in the future. This increases the value of existing matches, since with probability $1-\rho$ an existing match survives to the following period and eliminates the need to incur future job posting costs. An increase in the expected cost of future job postings lowers the effective cost of current labor matches. This fall in current labor costs when expected future labor market tightness rises reduces wholesale prices relative to retail prices. This reduces current retail price inflation. Finally, because it is the discounted value of expected future labor market conditions that affects the firm's decision to post an extra vacancy, there is a cost channel effect, as the real interest rate has a direct impact on ξ_t and therefore on inflation.

A rational expectations equilibrium satisfies (3), (4), the optimal retail pricing condition, (8) - (17), and the definitions of θ_t, q_t, p_t, and λ_t described in the text. These equations jointly determine Y_t, C_t, π_t, N_t, u_t, v_t, w_t, μ_t, θ_t, ξ_t, q_t, p_t, λ_t, r_t and the nominal interest rate i_t once a specification of monetary policy is added.

1.3 The linearized model

In this section, we derive a log-linear approximation of the rational expectations equilibrium around the efficient steady state. We then show that the log-linearized model can be reduced to a system of two equilibrium conditions that correspond to the new Keynesian expectational IS and Phillips curves, expressed in terms of unemployment and inflation rather than in terms of output and inflation.

Let $\hat{x}_t$ denote the log deviation of a variable x around its steady-state value $\bar{X}$, let $\hat{x}_t^e$ be the stochastic, efficient equilibrium value of $\hat{x}_t$, and let $\tilde{x}_t \equiv \hat{x}_t - \hat{x}_t^e$ denote the efficiency gap for $\hat{x}_t$, i.e., the gap between $\hat{x}_t$ and its stochastic, efficient equilibrium counterpart. In the first step, we use the goods market clearing condition, the production function, and the labor market conditions to express consumption in terms of unemployment. Goods market clearing requires that

$$Y_t = C_t - w^u\left(1-N_t\right) + \kappa V_t$$

as output is used for market consumption (total consumption minus home production, or $C_t - w^u(1-N_t)$) and vacancy posting costs. Log linearizing this condition yields

$$\hat{y}_t = \left(\frac{\bar{C}}{\bar{Y}}\right)\hat{c}_t + w^u\hat{n}_t + \left(\frac{\kappa\bar{V}}{\bar{Y}}\right)\left(\hat{\theta}_t - \hat{u}_t\right), \tag{18}$$

where use has been made of the fact that $\hat{\theta}_t = \hat{v}_t - \hat{u}_t$. From the CRS production function, $\hat{y}_t = \hat{n}_t + z_t$, so (18) implies

$$\left(\frac{\bar{C}}{\bar{Y}}\right)\hat{c}_t = (1-w^u)\hat{n}_t + z_t - \left(\frac{\kappa\bar{V}}{\bar{Y}}\right)\left(\hat{\theta}_t - \hat{u}_t\right) \tag{19}$$

Log linearizing (9), which links the number of employed workers and the number of job seeking workers, and (11), which governs the evolution of employment, yields

$$\hat{u}_t = -\eta\hat{n}_{t-1}, \text{ where } \eta \equiv (1-\rho)\left(\frac{\bar{N}}{\bar{u}}\right). \tag{20}$$

and

$$\hat{n}_t = (1-\rho)\hat{n}_{t-1} + \alpha\rho\hat{\theta}_t + \rho\hat{u}_t.$$

These two equations then imply

$$\hat{u}_{t+1} = \rho_u\hat{u}_t - \alpha\rho\eta\hat{\theta}_t, \tag{21}$$

where $0 < \rho_u \equiv (1-\rho)(1-\rho\bar{N}/\bar{u}) < 1$; higher labor market tightness reduces unemployment as more job seekers find employment matches. Combining (20) and (21) with (19) gives

$$\hat{c}_t = \varphi_1\hat{u}_{t+1} - \varphi_2\hat{u}_t + \left(\frac{\bar{Y}}{\bar{C}}\right)z_t, \tag{22}$$

where, $\varphi_1 \equiv -\left(\bar{Y}/\eta\bar{C}\right)\left[1 - w^u - \left(\kappa\bar{V}/\alpha\rho\bar{Y}\right)\right]$ and $\varphi_2 \equiv \left(\kappa\bar{V}/\alpha\rho\eta\bar{C}\right)(\alpha\rho\eta + \rho_u)$.

Since the representative household's optimal consumption plan will satisfy a standard log-linearized Euler condition, equation (22) can be used to eliminate $\hat{c}_t$ and obtain an Euler condition expressed in terms of the current and lagged number of job seekers, the real interest rate, and current and expected future productivity:

$$\hat{u}_{t+1} = \left(\frac{1}{\varphi_1+\varphi_2}\right)\left[\varphi_1 \mathrm{E}_t\hat{u}_{t+2} + \varphi_2\hat{u}_t - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t\pi_{t+1}) + \left(\frac{\bar{Y}}{\bar{C}}\right)(\mathrm{E}_t z_{t+1} - \hat{z}_t)\right]. \tag{23}$$

The appendix shows that at an efficient steady state, $\varphi_1/(\varphi_1+\varphi_2) = \beta/(1+\beta)$, so subtracting the flexible-price equilibrium conditions to express variables in terms of gaps, the Euler condition takes the form

$$\tilde{u}_{t+1} = \left(\frac{\beta}{1+\beta}\right)\mathrm{E}_t\tilde{u}_{t+2} + \left(\frac{1}{1+\beta}\right)\tilde{u}_t - \left(\frac{1}{\hat{\sigma}}\right)\tilde{r}_t; \tag{24}$$

where $\hat{\sigma} = \sigma(1+\beta)\left(\kappa\bar{V}/\alpha\bar{C}\right)\left(\alpha - 1 + \bar{U}/\rho\bar{N}\right)$.

In a standard NK model, the Euler condition is forward looking, containing no lagged endogenous variables. Often, the optimal monetary policy literature assumes habit persistence on the part of households, resulting in a lagged output gap term in the IS relationship. In our model, $\tilde{u}_t$, which is predetermined at time t, appears because search frictions cause equilibrium production to be affected by the number of workers who enter the period without matches or are displaced from existing matches. This leads to the presence of a backward-looking component in the IS relationship when expressed in terms of unemployment, even with standard household preferences. The weights on $\mathrm{E}_t\tilde{u}_{t+2}$ and $\tilde{u}_t$ in (24) are respectively $\beta/(1+\beta)$ and $1/(1+\beta)$, each approximately equal to one-half.

Given the Calvo-specification for price adjustment, the linearized Phillips curve takes the standard form given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \delta \hat{\mu}_t,$$

since the marginal cost for retail firms is μ_t^{-1}. Equation (16) implies $\hat{\mu}_t = z_t - \hat{\xi}_t$ and (17) can be linearized to allow $\hat{\xi}_t$ to be written in terms of current and expected labor market tightness, the real interest rate, and the bargaining disturbance. The retail price markup $\hat{\mu}_t$ then can then be expressed as

$$\begin{aligned} \hat{\mu}_t \quad = \quad & z_t - \hat{\xi}_t = z_t - A(1-\alpha)\hat{\theta}_t + A\beta(1-\rho)\left[1-\alpha-b\theta q(\theta)\right]\mathrm{E}_t\hat{\theta}_{t+1} \\ & -A\beta(1-\rho)\left[1-b\theta q(\theta)\right]\hat{r}_t - B\hat{b}_t, \end{aligned} \tag{25}$$

where $A \equiv \mu\kappa/(1-b)q(\theta)$, $B \equiv A[b/(1-b)]\left[1-\beta(1-\rho)(1-p)\rho_b\right]$, and we have assumed $\hat{b}_t$ follows an AR(1) process with serial correlation coefficient ρ_b. A rise in labor market tightness increases wages and reduces the retail price markup, increasing the marginal cost of retail firms. This leads to a rise in inflation. For a given $\hat{\theta}_t$, a rise in $\mathrm{E}_t\hat{\theta}_{t+1}$ increases the markup $\hat{\mu}_t$ and reduces current inflation.[8] Expectations of future labor market tightness implies a lower expected future job filling probability, raising the expected cost of filling a job in the future. This increases the value of an existing match and reduces the effective cost of labor (see 17). This fall in the labor costs of wholesale firms reduces wholesale prices relative to retail prices and reduces the marginal cost of retail firms. A rise in the real interest rate lowers the present value of the vacancy cost savings associated with an existing match, increases the effective cost of labor, and increases wholesale prices relative retail prices, leading to a rise in inflation. Finally, a rise in the bargaining power of workers raises labor costs and wholesale prices relative to retail prices, leading to a fall in the retail price markup.

To obtain a Phillips curve in terms of unemployment gaps, we use (21) to express $\hat{\theta}$ in terms of $\hat{u}_{t+1}$ and $\hat{u}_t$ and then use (25) to obtain

[8] In our baseline calibration, $1-\alpha-b\theta q(\theta) > 0$.

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \delta a_1 \left[\rho_u \beta \left(\mathrm{E}_t \tilde{u}_{t+2} - \rho_u \tilde{u}_{t+1}\right) - \left(\tilde{u}_{t+1} - \rho_u \tilde{u}_t\right)\right] / \alpha \rho \eta$$
$$+ \beta \delta a_3 \tilde{r}_t + \delta B \hat{b}_t, \tag{26}$$

where

$$a_1 = \left[(1-\alpha)/(1-b)\right] \left(\kappa \bar{V} / \rho \bar{N}\right)$$
$$a_2 = a_1 \left[(1-\rho)/(1-\alpha)\right] \left(1 - \alpha - \rho \bar{N}/\bar{u}\right)$$
$$a_3 = a_1 \left[(1-\rho)/(1-\alpha)\right] \left(1 - b\rho \bar{N}/\bar{u}\right).$$

A final simplification is obtained if (24) is used to eliminate $\mathrm{E}_t \tilde{u}_{t+2}$ from (26), yielding

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - a_1 \delta \left[\frac{(1-\beta\rho_u)(1-\rho_u)}{\alpha \rho \eta}\right] \tilde{u}_{t+1} + \delta \left[\beta a_3 + \left(\frac{a_1 \rho_u}{\sigma \alpha \rho \eta}\right)\right] \tilde{r}_t + \delta B \hat{b}_t. \tag{27}$$

Equation (27) is isomorphic to a NK Phillips curve with an unemployment rate gap replacing an output gap and with a cost channel present, though this latter channel operates through the real rate of interest rather than through the nominal rate as in Ravenna and Walsh (2006).[9]

2 Optimal monetary policy

To study optimal monetary policy, we assume the monetary authority's objective is to maximize the expected present discounted value of the utility of the representative household. A rich and insightful literature has developed from the initial contributions of Rotemberg and Woodford (1996) and Woodford (2003) employing policy objectives based on a second order approximation to the welfare of the representative agent. As is well known, the appropriate welfare approximation depends on the exact structure of the model. In this section, we discuss the quadratic objective function that arises in our model with sticky prices and labor market frictions. Mathematical details are given in an appendix available from the authors.

2.1 The quadratic approximation to welfare

Efficiency requires that three conditions hold: prices must be flexible so that the markup is constant; the fiscal subsidy τ must ensure the steady-state markup equals 1; and the Hosios (1990) condition

[9] Note that conditional on $\tilde{r}_t$ and $\tilde{u}_{t+1}$, the IS relationship (24) implies that $\tilde{u}_t + \beta E_t \tilde{u}_{t+2}$ must be constant. A higher value of $\tilde{u}_t$, again conditional on $\tilde{u}_{t+1}$, implies greater labor market tightness $\tilde{\theta}_t$, as vacancies must be higher to prevent the higher $\tilde{u}_t$ from leading to a rise in $\tilde{u}_{t+1}$. Greater labor market tightness in period t raises real marginal cost at t and would tend to increase inflation. But at the same time, $\beta E_t \tilde{u}_{t+2}$ must be lower to maintain $\tilde{u}_t + \beta E_t \tilde{u}_{t+2}$ constant, consistent with the Euler condition. The fall in $\beta E_t \tilde{u}_{t+2}$ implies an increase in expected future labor market tightness, and this acts to lower inflation. The two effects exactly offset leaving inflation independent of lagged unemployment.

must hold ($b = 1 - \alpha$).[10] The second order approximation to welfare when the steady state is efficient is

$$\sum_{i=0}^{\infty} \beta^i U(C_{t+i}) = \frac{U(\bar{C})}{1-\beta} - \frac{\varepsilon}{2\delta} U_c \bar{C} \sum_{i=0}^{\infty} \beta^i L_{t+i} + t.i.p. \tag{28}$$

where $t.i.p.$ denotes terms independent of policy, and the period-loss function is

$$L_t = \pi_t^2 + \lambda_0 \tilde{c}_t^2 + \lambda_1 \tilde{\theta}_t^2, \tag{29}$$

where $\lambda_0 = \sigma(\delta/\varepsilon)$ and $\lambda_1 = (1-\alpha)(\delta/\varepsilon)(\kappa\bar{V}/\bar{C})$. It is important to note that the weight on $\tilde{c}_t^2$ is the same as that obtained in a standard NK model if utility is linear in hours worked. That is, in the basic NK model, the relative weight on the output gap in the loss function is, in terms of the present notation, $\delta(\sigma + \eta_N)/(1 + \eta_N \varepsilon)\varepsilon$, where η_N is the inverse of the wage-elasticity of labor supply (see Woodford 2003 or Walsh 2010, p. 386). If $\eta_N = 0$, one obtains $\sigma\delta/\varepsilon$, which is the value of λ_0 in (29).

To understand this loss function, recall that in a standard NK model, utility is reduced by inefficient volatility of consumption, yet inflation also reduces utility because it generates a dispersion of relative prices that leads to an inefficient composition of consumption. That is, even if total consumption is equal to its efficient level, up to first order, the composition of consumption across individual goods is inefficient in the presence of inflation. Because of diminishing marginal utility with respect to leisure, inefficient fluctuations in hours also reduces welfare in the standard NK model. However, from the aggregate production function, hours can be expressed in terms of consumption so that loss can be written as a function of inflation volatility and consumption (output) volatility.

The standard distortion arising from inflation is also present in the model with labor search frictions. Therefore, as in the NK model, welfare is decreasing in inflation volatility. And because of diminishing marginal utility, volatility of consumption reduces welfare. The marginal disutility of working is constant in our framework, but to transfer workers from home production to market production involves the matching function, which is characterized by diminishing marginal productivity with respect to labor market tightness as long as $0 < \alpha < 1$. Since market consumption is output less vacancy posting costs, the costs of job posting rise more when vacancies increase than they fall when vacancies decrease. Thus, volatility in vacancies relative to their efficient level reduces welfare and accounts for the separate term in labor market tightness that appears in the loss function (29).[11]

Even if inflation is zero, so that market consumption is obtained through an efficient combination of the differentiated market goods, the composition of total consumption between market goods and home production can be inefficient if vacancy postings, and thus the wedge between output

[10] See the appendix for the proof of this statement.

[11] Search frictions also affect the equilibrium movements of the consumption gap by changing the propagation mechanism and thus optimal policy. The change in the propagation mechanism does not, however, result in a change on the *weight* on the consumption gap in the loss function.

allocated to consumption and to cover the cost of search, deviates from the efficient value. This result does not hinge on our particular specification of home production or search costs (as long as they are not linear) but simply on the fact that an alternative way of generating utility (home production) is available to unemployed agents, and this alternative does not suffer from the search friction necessary to produce matches and market consumption. This source of inefficient resource allocation would continue to be present if the model were extended to allow for variable hours in production and disutility of hours worked.

In (29), the weight on $\tilde{\theta}$ gap volatility relative to consumption gap volatility is equal to $(1-\alpha)\kappa\bar{V}/\bar{C}$. Rewriting this as $(1-\alpha)\left(\bar{C}^m/\bar{C}\right)\left(\kappa\bar{V}/\bar{C}^m\right)$ shows that as vacancy costs associated with producing market consumption rise or market consumption's share of total consumption rises, the welfare cost of θ-gap fluctuations increases. From the matching function, α is the elasticity of the value of a filled job with respect to θ; if $\alpha = 1$, the matching technology displays constant returns to scale with respect to θ and volatility in $\tilde{\theta}$ does not generate a welfare loss. The additional resources devoted to posting vacancies when $\tilde{\theta} > 0$ are exactly offset by the lower costs when $\tilde{\theta} < 0$. However, for $0 < \alpha < 1$, the matching function is characterized by decreasing returns to θ. The additional costs of vacancies when $\tilde{\theta} > 0$ exceeds the cost savings that occur when $\tilde{\theta} < 0$, and the overall welfare loss from volatility in $\tilde{\theta}$ is greater when $1-\alpha$ is large. Changes in α will also affect the steady-state cost of vacancy posting relative to consumption. A rise in the elasticity of matches with respect to vacancies (a rise in α) increases the level of vacancies in the steady state and leads to a rise in $\kappa\bar{V}/\bar{C}$. This acts to increase the welfare cost of volatility in the $\tilde{\theta}$-gap. Whether a rise in α increases or decreases the cost of inefficient fluctuations in labor market tightness will depend on the calibration of the model's parameters. We return to this point in the following section after discussing our baseline calibration.

In a similar model, Thomas (2008) derives a second order approximation to the utility of the representative agent that consists of a term that is quadratic in inflation, reflecting the loss from price dispersion, and additional terms made up of squares of a number of endogenous variables, including consumption, employment, and labor market tightness. This second term cannot be written in terms of variables measuring *gaps* relative to the efficient equilibrium, so it does not provide a way to disaggregate the inefficiencies created by the search frictions from those created by nominal price stickiness. In contrast, our approximation expresses the loss function in terms of inefficiency gaps that the policy maker would want to minimize and provides the weights that the policy maker should attach to each inefficiency gap.

Because policy concerns about the labor market are normally expressed in terms of unemployment, and not labor market tightness, it is useful to replace the θ-gap in the loss function using (21). Making this substitution, the quadratic loss function becomes

$$L_t = \pi_t^2 + \lambda_0\tilde{c}_t^2 + \bar{\lambda}_1\left(\tilde{u}_{t+1} - \rho_u\tilde{u}_t\right)^2, \tag{30}$$

where $\bar{\lambda}_1 = \lambda_1(1/\alpha\rho\eta)^2 = (1-\alpha)\left(\delta/\varepsilon\right)\left(\kappa\bar{V}/\bar{C}\right)(1/\alpha\rho\eta)^2$. Both $\tilde{u}_{t+1}$ and $\tilde{u}_t$ matter because of the persistence exhibited by employment matches. If all matches dissolved at the end of every period as in a standard NK model, so that $\rho = 1$ and $\rho_u = 0$, log-linearization of (9), (10), and (11) implies $\hat{n}_t = \alpha\hat{\theta}_t$. With $\hat{n}_t$ and $\hat{\theta}_t$ moving proportionally, the consumption gap and labor market tightness gap could be combined into a single consumption gap variable. When matches persist (i.e., when $\rho < 1$), current employment depends on current labor market tightness, but it also depends on the stock of matches that survived from the previous period.

Since (22) implies $\tilde{c}_t = \varphi_2 (\beta\tilde{u}_{t+1} - \tilde{u}_t)$,[12] we could also write the loss function in the form

$$L_t = \pi_t^2 + \bar{\lambda}_0 (\beta\tilde{u}_{t+1} - \tilde{u}_t)^2 + \bar{\lambda}_1 (\tilde{u}_{t+1} - \rho_u\tilde{u}_t)^2$$

with $\bar{\lambda}_0 = \lambda_0\varphi_2^2$. If the initial unemployment gap $\tilde{u}_t$ is zero, maintaining $\tilde{u}_{t+i} = 0$ for all $i > 0$ also ensures that $\tilde{c}_{t+i} = 0$ for all $i \geq 0$. However, if $\tilde{u}_t \neq 0$, then the central bank must trade-off efficient labor market tightness – which would require setting $\tilde{u}_{t+1} = \rho_u\tilde{u}_t$ – against volatility in the unemployment gap – which would call for setting $\tilde{u}_{t+1} = \tilde{u}_t/\beta$.

With our baseline calibration, which is discussed in the following section, λ_1 is small, reflecting in part the fact that vacancy costs are small relative to total output. In fact, if we assume terms of the form $(\kappa\bar{V}/\bar{N})\hat{x}_t\hat{y}_t$ are third order, then the loss function for a second-order approximation to welfare would take the form

$$\pi_t^2 + \lambda_0\tilde{c}_t^2 \tag{31}$$

and involve only inflation and the consumption gap.[13] However, when expressing loss in terms of the unemployment gap as in (30), $(1/\alpha\rho\eta)^2$ is approximately equal to 13 under our baseline calibrations, so even when λ_1 is small, we do not drop this term when we derive optimal policy.

2.2 Responses under optimal monetary policy

In this section, we examine equilibrium under the optimal timeless perspective form of commitment policy (Woodford 2003), assuming the central bank acts to minimize the loss function given by our quadratic approximation to welfare. The constraints on policy are given by (24) and (26). Since the productivity shock does not appear explicitly in either the objective function or the constraints of the policy problem, optimal policy insulates inflation and the unemployment gap from productivity shocks and lets actual unemployment move with the efficient, flexible-price unemployment rate. The central bank simply adjusts policy to keep the real interest rate gap $\tilde{r}_t$ equal to zero whenever productivity shocks move the efficient level of the real interest rate. This result, however, is the consequence of our assumption that the Hosios condition holds in the steady state so that wage setting under Nash bargaining yields the efficient outcome. If b were to differ from $1 - \alpha$, a productivity shock would present the policy maker with a trade-off between moving the interest rate so as to stabilize inflation or moving the interest rate to steer firms' incentive to post vacancies towards the efficient level.

[12] In the efficient steady state, $\varphi_1 = -\beta\varphi_2$ (see the appendix).

[13] BG also assume hiring costs are small, leading them to drop cross-product terms with hiring costs, so (31) would represent the loss in our model under assumptions similar to those used by BG.

Even when b equals $1-\alpha$ on average, stochastic fluctuations in bargaining shares present the central bank with a trade-off between stabilizing inflation and stabilizing variability in the unemployment gap. A positive realization of $\hat{b}_t$ pushes up wages and the price of wholesale goods relative to retail goods. This increases the marginal costs of the retail firms and leads to a rise in inflation. It also leads wholesale firms to post fewer vacancies, leading to a decline in employment. The policy maker would want to raise interest rates to offset the inflationary impact of this shock, but doing so worsens the decline in labor market tightness through a standard aggregate demand channel and, from (12), by reducing the present value of a filled match.[14] Essentially, the bargaining shock enters (27) as a cost-push shock since it is associated with inefficient fluctuations in unemployment. When bargaining shares fluctuate, stabilizing inflation and stabilizing labor market variables become conflicting objectives even if the initial unemployment gap is zero.

Our approach does not take a stand on the sources of these fluctuations in $\hat{b}_t$ and simply assumes they are exogenous. Other micro-founded policy objective functions make stronger assumptions on the source of the inefficiency by modeling explicitly deviations of the wage and of the surplus share from the efficient equilibrium. Thomas (2008), for example, assumes staggered wage adjustment for both new and incumbent workers. Clearly, we could replicate any endogenous wage sequence generated by a productivity shock by building an appropriate sequence of $\hat{b}_t$ shocks.[15]

To evaluate policy outcomes, we calibrate the model. The baseline values for the model parameters are set to standard values in the literature and are given in Table 1. We assume the period length is one quarter and set $\beta = 0.99$. We impose the Hosios condition in the steady state by setting $b = 1-\alpha$. Estimates of α, the elasticity of matches with respect to vacancies, generally fall in the 0.4 to 0.6 range, so we set $\alpha = 0.5$. The U.S. unemployment rate averaged 5.84 percent over the 1983-2007 period, so we set steady-state employment to equal $1-0.0584 = 0.9416$. We calibrate the replacement ratio $\phi \equiv w^u/w$ at 0.54 for the U.S. based on estimates from the OECD database on benefits and wages. From the estimated monthly separation rate of 3.4 percent reported in Shimer (2005), we set the quarterly separation rate ρ equal to 10 percent.[16] This is consistent with the estimates of Davis, Haltiwanger, and Schuh (1996) and is the value employed in the related literature.[17] Estimates of the steady-state job finding probability q vary widely in the literature. den Haan, Ramey, and Watson (2000) cite data from Davis, et.al. (1996) to calibrate q equal to 0.71. Cooley and Quadrini (1999) and Walsh (2005) also set $q = 0.7$. This value may be too low, as Davis, Faberman, and Haltiwanger (2009) estimate a daily job-filling probability of around 5 percent. Following their assumption of an average of 26 working days per month, or three times

[14] From (12),

$$J_t = \left(\frac{P_t^w}{P_t}\right) Z_t - w_t + (1-\rho)\mathrm{E}_t\left(\frac{1}{R_t}\right) J_{t+1}$$

so conditional on the wage, an increase in the real interest rate reduces J_t. Equation (17) shows how the effective cost of labor is increasing in the real interest rate.

[15] Given the ongoing debate on the most appropriate way to describe wage setting, and the ambiguous evidence on wage rigidity for new hires (Haefke et. al., 2007, Pissarides 2009), our approach provides a reasonable and useful benchmark.

[16] We translate the monthly rate into a quarterly rate following the method of Blanchard and Galí (2010).

[17] For example, den Haan, Ramey, and Watson (2000), Walsh (2003, 2005), and Blanchard and Galí (2010).

that per quarter, a daily rate of f would imply the probability a vacancy is filled over a quarter to be roughly $1-(1-f)^{3*26}=0.98$.[18] Because we use steady-state conditions to solve for the job posting cost κ and the wage w, variations in q have little effect on our results, so we set $q=0.9$.[19]

The volatility of the bargaining and productivity shocks are chosen so that, conditional on a policy of price stability, the standard deviation of output is $\sigma_{Y_t}=1.82$ percent and the standard deviation of employment is $\sigma_{N_t}=1.71$ percent. These values are in line with U.S. business cycle dynamics over the postwar period, and result in a volatility of the innovations for $\hat{b}_t$ and $\widehat{Z}_t$ of 3.87 percent and 0.32 percent and an output-to-employment volatility equal to 0.94. The high ratio between the volatility of the bargaining and productivity shocks needed to match the data is an implication of the well known Shimer puzzle: in a search labor market model of the business cycle with flexible wages and constant surplus shares, the relative volatility of unemployment and output would be one order of magnitude too small. In a model where the wage adjustment process results in time-varying surplus shares, bargaining shocks would not be needed in order to match the data. We assume a first order autocorrelation coefficient of 0.8 for both exogenous shocks.[20] Sala, Söderström and Trigari (2008) show that, in an estimated DSGE model with labor market search frictions where the policy maker aims at stabilizing π_t and $\tilde{u}_t$, markup shocks generate a much more severe trade-off between stabilizing inflation and the unemployment gap, compared to bargaining shocks. This is especially so when wages are flexible, a case where, as in our model, technology shocks generate no trade-off. In our model, bargaining shocks are the only cost-push shocks; therefore, they are responsible for all the deviations of unemployment and inflation from the efficient equilibrium under the optimal policy.

The behavior of inflation and unemployment in the face of a bargaining shock under optimal policy will depend on the relative weights on the policy objectives. These weights, in turn, depend on the parameters such as α, the elasticity of matches with respect to vacancies, and ρ, the rate of exogenous job destruction, that characterize the labor market. Figures 1 and 2 provide some evidence on how these parameters affect the trade offs between policy objectives. Figure 1 shows $\lambda_1/\lambda_0=(1-\alpha)\kappa\bar{V}/\bar{C}$, the weight on the labor market tightness gap relative to the consumption gap in the loss function (29) as a function of α and ρ. As noted earlier, α has two effects on this ratio. First, an increase in α reduces the convexity of matches as a function of labor market tightness and lowers the cost of inefficient fluctuations in $\tilde{\theta}$; this is the role of the $1-\alpha$ term. Second, an increase in α raises steady-state job posting costs relative to consumption $\kappa\bar{V}/\bar{C}$ and this increases the cost of inefficient fluctuations in $\tilde{\theta}$. As figure 1 illustrates, this second effect dominates and λ_1/λ_0 is

[18] This simple calculation ignores that fact that some vacancies within a quarter are closed without being filled and new vacancies are posted within the quarter. Davis, Faberman, and Haltiwanger (2009) account for these flows in obtaining their daily job-filling rate.

[19] Results for alternative values of q are available from the authors. To find κ and w, assume $w^u=\phi w$, where ϕ is the wage replacement rate. Then (13) and (15) can be jointly solved for κ and w.

[20] For the U.S. nonfarm business sector, the volatility of HP filtered GDP over the 1955-2006 period is equal to 2.05 percent; the volatility of the detrended employed to civilian non-institutional population ratio is 1.41 percent., and the volatility of detrended total per capita labor hours (computed as average weekly hours for private industries multiplied by the employed to civilian non-institutional population ratio) is 3.01%. Source: BLS and Federal Reserve ALFRED database.

increasing in α. In our calibration, we impose the Hosios condition that labor's share of the match surplus equals $1-\alpha$ to ensure the steady-state equilibrium is efficient. Hence, as α increases, labor's share of the match surplus falls. Thus, the costs of labor market volatility increase as labor's share of the surplus declines. This cost also increases with ρ, the rate of exogenous job separation as greater turnover raises the steady-state value of $\kappa\bar{V}/\bar{C}$.

As shown earlier, the θ-gap could be replaced in the loss functions by $\left(\rho_u \tilde{u}_t - \tilde{u}_{t+1}\right)/\alpha\rho\eta$ so that the quadratic loss function becomes

$$\pi_t^2 + \lambda_0 \tilde{c}_t^2 + \bar{\lambda}_1 \left(\tilde{u}_{t+1} - \rho_u \tilde{u}_t\right)^2.$$

To compare the weight on the consumption gap to the weight on unemployment, figure 2 plots $\bar{\lambda}_1/\lambda_0$ as a function of α and ρ. While stabilizing the unemployment term becomes more important relative to stabilizing consumption as ρ increases, it becomes less important as α increases. This may seem at odds with the results in figure 1, but as α increases, volatility in $\tilde{u}_{t+1} - \rho_u \tilde{u}_t$ is associated with less volatility in $\tilde{\theta}_t$. Thus, even though an increase in α calls for stabilizing labor market tightness more, as shown in figure 1, this can actually be achieved with greater volatility in $\tilde{u}_{t+1} - \rho_u \tilde{u}_t$.

The dynamic responses of inflation, the unemployment gap, and $\tilde{\theta}_t$ to a one standard deviation innovation to the bargaining shock under optimal policy for the parameters in Table 1 are shown in figure 3 for both a serially uncorrelated process (i.e., $\rho_b = 0$) and a persistent one ($\rho_b = 0.8$).[21] The rise in labor's share due to the positive shock pushes up costs and leads to a rise in inflation. It also leads to an inefficient drop in vacancies and rise in the unemployment gap. Labor market tightness declines. This is the result of both a decrease in the job finding probability and an increase in the probability of filling a vacancy. The shock to the bargaining share generates dynamic behavior akin to a cost-push shock in the NK model, where output is below the efficient level and inflation is positive on impact. The dynamic process of adjustment in the labor market leads to a gradual return of unemployment to its efficient level. *Under the optimal policy*, the impact of the shock on inflation is small; inflation rises by only 3 basis points at an annual rate.

2.3 The role of the loss function

In this section, we investigate the consequences of policies that are optimal for a mis-specified objective function. In particular, we consider the welfare costs of designing policies to minimize an objective function that corresponds to the quadratic loss functions commonly employed in the literature on optimal monetary policy. We consider two alternatives to the welfare-based loss function. The first alternative simply drops the $\tilde{\theta}_t^2$ term, yielding a loss function that more closely parallels a standard quadratic loss function:

$$L_t^{nk} \equiv \pi_t^2 + \lambda_0 \tilde{c}_t^2. \tag{32}$$

[21] Since standard deviation of $\hat{b}_t$ is 3.87 percent, the positive innovation corresponds to a shock that changes b_t from its steady-state value of 0.5 to $0.5 \times 1.0387 = 0.5193$.

In this case, policy aims to stabilize inflation volatility and the volatility of the consumption gap. We employ the welfare-based value of λ_0 since, as noted earlier, this is equal to the same value that would arise in a standard NK model in which utility depends linearly on hours worked. This loss function ignores the inefficiencies arising from search costs in the labor market.

A second loss function previously employed in the literature includes inflation and the unemployment rate gap:

$$L_t^u(\lambda) \equiv \pi_t^2 + \lambda \tilde{u}_t^2. \tag{33}$$

Such a loss function has been employed by Orphanides and Williams (2007) and is used by Sala, Söderström, and Trigari (2008) in a model with search and matching frictions in the labor market. Because (33) represents an ad-hoc specification of policy objectives, theory offers no guidance as to the value to assign to λ, the relative weight placed on unemployment objectives. For our baseline, we set λ so that the standard deviation of the unemployment gap under commitment is the same when minimizing either (33) or the welfare-based loss function (29). In this case, $\lambda = 0.0035$. Sala, Söderström, and Trigari (2008) derive optimal policy for various values of λ and find that a value of 0.0521 matches the standard deviation of unemployment in their model.[22] Therefore, we also report results for $\lambda = 0.0521$. Since this value of λ is nearly 15 times the one that would deliver the same unemployment gap volatility as the optimal policy, it will imply a very high volatility of inflation in our model. This experiment is useful though to provide a measure of the sensitivity of the loss to the relative weight placed by the policy maker on competing objectives. Orphanides and Williams (2007), for example, employ an even larger weight of 0.25 on unemployment in their analysis.

Results when policy is based on minimizing (under commitment) the alternative loss functions (32) and (33) are reported in Table 2. The first column of the table reports the percentage increase in the welfare-based loss function given by (29) when policy minimizes one of the alternative loss functions. Minimizing (32), for example, increases the loss by 4.59 percent. When policy minimizes inflation and unemployment volatility, the weight placed on the unemployment gap is crucial; minimizing (33) increases the loss by 0.34 percent when $\lambda = 0.0035$ but by 275.93 percent when the value $\lambda = 0.0521$ is used.

To measure the welfare loss in terms of steady-state consumption, note that in (28) only the term involving the quadratic loss L_{t+i} differs under alternative policies. Let

$$L^p \equiv -\frac{\varepsilon}{2\delta} U_c \bar{C} \mathrm{E} \sum_{i=0}^{\infty} \beta^i L_{t+i}^p$$

denote the loss under a policy p and

$$L^r \equiv -\frac{\varepsilon}{2\delta} U_c \bar{C} \mathrm{E} \sum_{i=0}^{\infty} \beta^i L_{t+i}^r$$

[22] Because they express inflation at an annual rate, the actual value of λ they use is $16 \times 0.0521 = 0.833$.

the loss under the welfare-based optimal commitment (Ramsey) policy. The welfare cost of implementing policy p can be measured as the percent increase in steady-state consumption λ^p that would make the representative agent indifferent between policy p and the Ramsey policy, where λ^p solves

$$\frac{U((1+\lambda^p)\bar{C})}{1-\beta} + L^p + t.i.p. = \frac{U(\bar{C})}{1-\beta} + L^r + t.i.p.$$

The resulting measure is reported in column two of Table 2. Consistent with the comparison based on the quadratic loss itself, the welfare costs of deviating from the optimal commitment policy are small in terms of steady-state consumption equivalents except when a large weight is placed on the volatility of the unemployment gap. In fact, when $\lambda = 0.0521$ in (33), performance deteriorates significantly (see row 4, Table 2). With this parameterization, policy is much more aggressive in stabilizing deviations of unemployment from the efficient level; the standard deviation of inflation increases by a factor of eight, while the standard deviation of the unemployment gap falls by about one third. The monetary authority would do much better by focusing on stabilizing inflation and ignoring altogether the impact of bargaining shocks on employment, as the second row of Table 2 shows.

The responses of inflation, the unemployment gap and labor market tightness to a one standard deviation, serially correlated bargaining shock for the different policy objectives are shown in figure 4. For comparison, the lines marked by circles give the impulse responses under the welfare-based optimal commitment policy and are the same as those shown in figure 3. The responses are quite similar across the different loss functions with the exception of (33) when a large weight is placed on unemployment fluctuations. This loss function allows a much greater response of inflation to the bargaining shock and, correspondingly, allows much less movement in the labor market variables. The policy based on the consumption gap loss given by (32) allows the most labor market volatility and almost completely neutralizes the impact of the bargaining shock on inflation. Both the welfare-based policy and the policy that minimizes (33) with $\lambda = 0.0035$ produce almost identical impulse responses in reaction to the bargaining shock.

2.4 Discretion versus commitment

In this section, we examine outcomes when policy is conducted in a discretionary regime. Results are reported in Table 3, which parallels the cases considered in Table 2 for optimal commitment. Several points are worth noting. First, the welfare cost of bargaining shocks under optimal discretion is 10.5 percent higher than obtained under the optimal commitment policy. This cost arises primarily from greater volatility of inflation under discretion. In fact, labor market outcomes are quite similar under either commitment or discretion, as shown in figure 5 which compares the impulse responses under the two policies. The path of inflation differs under commitment and discretion primarily because of the different paths followed by expected inflation under the alternative policy regimes. This is illustrated in figure 6 which decomposes the path of inflation, net of the direct effect of the bargaining shock, into the contributions of the labor market variables, expected inflation, and the real interest rate cost channel. Finally, under discretion the policy obtained using the standard NK model objective function results in an allocation closer to the commitment case and generates a higher welfare than the optimal discretionary policy.

Comparing Tables 2 and 3 shows that outcomes are fairly similar under either commitment or discretion except when policy minimizes a loss function that puts a large weight on unemployment gap variability. In this case, loss under discretion deteriorates significantly relative to policy based on the correct welfare approximation, rising from 0.0683 percent of steady-state consumption to 0.4815 percent. Optimal discretion based on (33) with a large weight on unemployment gap fluctuations results in the welfare-based loss function being almost 2000 percent higher than is achieved under optimal commitment. The increase in loss occurs because discretion smooths labor market variables to a much greater degree than is done under commitment with a corresponding increase in the volatility of inflation. Figure 7 shows that, while the immediate impact of rising unemployment on inflation is larger under discretion than commitment, the labor market returns to steady state much faster under discretion. As a consequence, expected inflation remains higher under discretion.

2.5 The role of the transmission mechanism

Above we assumed policy was optimal, conditional on the wrong objective. That tells us how important deviations from the correct objective function are in generating welfare losses relative to the optimal plan (Ramsey allocation). Such an exercise is silent, however, on the implications of an optimal policy that is conditional on the wrong constraints - that is, on the wrong transmission mechanism for policy. In this section, we investigate the performance of targeting rules that are optimal for the standard NK model. Since a central bank is likely to target the wrong objective if its knowledge of the transmission mechanism is inaccurate, we examine the performance of policy rules that would be optimal conditional on an incorrect objective function *and* on an incorrect transmission mechanism. This exercise provides insights into whether the central bank generates large losses by ignoring the existence of search frictions and search unemployment.[23]

We consider five alternative policy rules that have been widely employed in standard NK models: 1) the optimal targeting rule under commitment from a timeless perspective – $\pi_t = -(\lambda/\delta)(\tilde{c}_t - \tilde{c}_{t-1})$ where $\lambda = \delta(\sigma + \eta_N)/(1 + \eta_N \varepsilon)\varepsilon$ and $\eta_N = 1$ is the parameterized labor supply elasticity in a model with separable utility in consumption and labor hours; 2) the optimal targeting rule under the time-consistent, discretionary policy – $\pi_t = -(\lambda/\delta)\tilde{c}_t$; 3) strict inflation targeting – $\pi_t = 0$; 4) a Taylor rule – $i_t = 0.5\tilde{c}_{t-1} + 1.5\pi_t$;[24] and 5) a policy rule estimated by Clarida, Galí and Gertler (2000) over the Volcker-Greenspan years 1979-1997 –

[23] To provide a measure of how sensitive the loss is to alternative policies in a given economic environment, we keep constant all parameters, including the exogenous shocks' volatility, implying the resulting volatility of the endogenous variables does not necessarily match the data we used in our calibration.

[24] Because inflation and the interest rate in our model are expressed at quarterly rates, we divide the coefficient on $\tilde{c}_t$ byt 4 in the actual simulations. However, we present the Taylor rule here in its more familar form since we always express inflation at annual rates in the tables and figures. The Taylor rule is written in terms of lagged, rather than current consumption, to ensure determinacy of the equilibrium; see Kurozumi and van Zandwedge (2008) for determinacy conditions in a model akin to our specification.

$i_t = 0.71 i_{t-1} + 0.29\,(1.72\mathrm{E}_t \tilde{c}_{t+1} + 0.3\mathrm{E}_t \pi_{t+1})$.

Implementing the optimal commitment targeting rule from the standard NK model leads to a 6.62 percent increase in the welfare loss (see the first column of Table 4, row 1) relative to the commitment policy based on minimizing the welfare-based loss function. Interestingly, strict inflation targeting outperforms both the commitment and time-consistent targeting rules. This simply reflects the fact that when policy employs the wrong model, there is no presumption that an optimized policy will lead to better outcomes. In some NK frameworks, such as the Erceg, Henderson and Levin (2000) staggered wages-staggered prices model, or the baseline NK model with no cost-push shock in Galí (2002), a simple Taylor rule performs nearly as well as the optimal policy. This is not the case when search frictions generate a welfare cost. The third row of Table 4 shows that the Taylor rule does very poorly, leading to much more inflation variability and a loss more than 1200 percent larger than achieved under the optimal policy. The policy rule estimated by Clarida, Galí and Gertler (2000) over the Volcker-Greenspan years 1979-1997 leads to an even worse result (row 4), generating a welfare loss equal to 1.94 percent of steady state consumption relative to the Ramsey policy.

3 The role of labor market structure

An important advantage of our model is its ability to characterize the optimal policy according to different assumptions about the characteristics of the labor market. For example, Blanchard and Galí (2010) argue that Europe is characterized by a lower separation rate and level of steady-state employment than the U.S. They suggest setting $\rho = 0.025$ to capture the lower turnover in Europe. Over the period 1983-2007, Euro Area unemployment has averaged 10.11%, so we set $N = 1 - 0.1011 = 0.8989$. based on the OECD evidence on average unemployment benefit replacement ratios for Euro area countries, we set $\phi = 0.65$.[25]

Based on these calibration differences, the inflation adjustment equation for the U.S. expressed in the form given by (27) is

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - 0.087\tilde{u}_{t+1} + 0.103\tilde{r}_t + 0.081\hat{b}_t \tag{34}$$

while that for the EU calibration is

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - 0.065\tilde{u}_{t+1} + 0.845\tilde{r}_t + 0.099\hat{b}_t. \tag{35}$$

Two differences are apparent. First, the interest rate channel on inflation is much larger in the EU calibration. Second, inflation is less sensitive to the unemployment gap under the EU calibration.

[25] This is in line with Nickell (1999) who reports an average replacement ratio for EU countries of 0.6 and 0.5 for the U.S. The average duration of unemployment benefits is also shorter in the U.S. than in EU countries. It is important to note that the differences between our U.S. and EU calibrations is restricted *only* to different values of the labor market parameters. Other parameters, including the frequency of price adjustment, are held constant across the two calibrations.

In large part, these effects reflect the higher persistence of unemployment under the EU calibration, for which $\rho_u = 0.798$ versus a value of only 0.345 under the U.S. calibration. The higher degree of persistence reduces the impact of $\tilde{u}_{t+1}$ on inflation. If ρ_u is large, both current and future labor market conditions move together, so the impact of current conditions is offset to some degree by the co-movement of expected future conditions. In more flexible labor markets, ρ_u is smaller and current unemployment conditions induce a smaller co-movement in expected future conditions. The greater persistence of matches also explains why changes in the interest rate, which affect the present value of the match surplus, have a bigger impact under the EU calibration (see 26). When employment is more persistent, the expected discounted future labor market conditions have a bigger impact on current inflation (see 17), so changes in the rate used to discount the future have a correspondingly larger impact on current inflation.

Figure 8 plots the responses to a serially correlated shock to labor's bargaining share for the U.S. and EU calibrations. To allow the two cases to be more easily compared, the middle panel shows the response of the *level* of the standard unemployment rate, as steady-state levels differ under the two calibrations. The EU calibration leads to less volatility in the inflation rate and in labor market variables. This greater stability is the result of two factors. First, λ_1, the weight on labor market tightness in (29), is equal to $(1-\alpha)(\delta/\varepsilon)(\kappa\bar{V}/\bar{C})$. With the EU calibration reflecting a less flexible labor market, the steady-state share of output devoted to vacancy posting costs is smaller, reducing the welfare loss from inefficient fluctuations in the labor market and resulting in a smaller λ_1.[26] To understand why the reduction in the rate of exogenous job destruction ρ from 0.1 under the U.S. calibration to 0.025 for the EU calibration leads to greater stabilization of inflation, consider the limit as $\rho \to 0$. With no employment turnover and no vacancies, unemployment is constant, and optimal policy reduces to stabilizing the inflation rate at zero. Thus, optimal policy under the EU calibration assigns relatively more weight to achieving inflation stability than would be the case under the US calibration. Second, the effect of the interest rate on inflation is much stronger under the EU calibration, and this improves the inflation-unemployment trade-off. When matches persist with greater probability, as is the case under the EU calibration, the expected savings in future vacancy costs from having an existing match is larger, and changes in the interest rate that affect the present value of these savings have a bigger impact on the effective cost of labor. A positive bargaining shock increases inflation and reduces unemployment, but reducing the interest rate to boost employment through the standard demand channel also acts to significantly reduce inflation through its direct effect on labor costs, improving the inflation-unemployment trade-off.

Results for alternative loss functions with the EU calibration are given in Table 5. Regardless of the objective function, both inflation and the unemployment gap are significantly less volatile when the labor market is less flexible. Comparing the last column of Table 5 with the corresponding column of Table 2 shows that the relative decline in inflation volatility is greater, however. As under the U.S. calibration, the last row of Table 5 shows that a policy that places a large weight

[26] Since λ_0, the weight on the consumption gap, is independent of ρ, figure 1 implies that the weight placed on stabilizing labor market gaps falls as ρ declines.

on unemployment volatility in the loss function leads to a significant deterioration in welfare. Outcomes when policy minimizes the standard loss function (32) with $\lambda = 0.0521$ are shown in figure 9. In this case, the inflation response increases by nearly an order of magnitude under both calibrations (compare scales in figures 8 and 9), while the change in the behavior of labor market variables is much more modest. Under discretion, the adoption of the inefficient policy leads to drastically different outcomes for the EU and U.S. calibration: the costs of bargaining shocks is still relatively small in absolute value for the EU case, resulting in a loss of 0.06 percent of steady-state consumption, while it rises to 0.48 percent of steady-state consumption for the U.S. calibration.

4 Conclusions

A growing literature has incorporated labor market frictions into models with nominal rigidities. When these frictions are modelled using the search and matching approach of Mortensen and Pissarides, optimal policy has generally been studied using ad-hoc loss functions of the form commonly employed in models that lack labor market frictions. We derive an explicit second-order approximation to the welfare of the representative agent in the presence of search and matching frictions. The resulting model-consistent loss function includes an additional quadratic term involving labor market tightness that is missing from the standard loss function. This extra term captures the welfare loss due to search frictions when labor market tightness deviates from its efficient level. Thus, all the cost from search inefficiency can be summarized in a single term in the welfare function. This term can also be expressed solely in terms what we have labeled the unemployment gap, the gap between unemployment and the efficient level of unemployment.

Just as inflation creates an inefficiency in the way market goods are combined to generate market consumption, volatility in the labor market tightness gap generates inefficiency in the way market and home consumption are combined to generate overall utility. While we focus on search frictions, this results suggests that any distortion in the allocation of time among alternative uses that generate utility would lead to a labor market term in the loss function. In that sense, the basic results are not special to search frictions, though of course the exact form of the loss would depend on the nature of the distortion.

The trade-off between stabilizing the terms in the welfare function results from random deviations from efficient surplus sharing across firms and workers. Our approach does not take a stand on the sources of these fluctuations, and does not assume any endogenous constraint affecting wage adjustment, exactly as in the standard NK model. By preserving the assumption of real wage flexibility, our results are directly comparable to the models with staggered price adjustment and Walrasian labor markets with flexible wages that have been prominent in monetary policy analysis. However, our framework allows for a rich characterization of the labor market in terms of exogenous separation rates, labor bargaining power, matching technology productivity and efficiency, and job posting costs. We find, for example, that if the labor market becomes less flexible, as is generally assumed to be the case for European labor markets, employment dynamics are more muted in response to random shocks and optimal policy calls for greater inflation stability.

Ignoring the role of labor market frictions in setting the objectives of policy can lead to large losses. In particular, a policy designed to minimize inflation volatility and unemployment gap volatility can lead to a significant reduction in welfare if a moderately large weight is placed on unemployment objectives. If the central bank ignores the impact of labor market frictions in both the objective function and the transmission mechanism and implements the optimal targeting rule derived from a standard NK model, outcomes are inferior to those obtained under strict inflation targeting.

In standard NK models, cost shocks generate a wedge between the marginal rate of substitution between leisure and consumption and the marginal product of labor, producing a welfare cost that calls for deviating from price stability. With long-term employment relationships and efficient bargaining, temporary fluctuations in bargaining shares, the source of cost shocks in our model, do not generate large welfare costs since workers and firms are concerned with the present value of the match surplus. This is why price stability turns out to remain close to the optimal policy.

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Table 1: Parameter Values		
Exogenous separation rate	ρ	0.1
Vacancy elasticity of matches	α	0.5
Replacement ratio	ϕ	0.54
Steady state vacancy filling rate	q	0.9
Labor force	N	0.9416
Discount factor	β	0.99
Relative risk aversion	σ	2
Markup	μ	1.2
Price adjustment probability	$1-\omega$	0.25

Table 2: Alternative Policy Objectives						
	Commitment					
	Quadratic loss Relative to Opt. Commitment (%)	Welfare-based loss Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(1)	0	0	0.24	0.72	11.82	0.33
	Std. Loss in π and $\tilde{c}-$gap, $\lambda=\lambda_0$					
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(2)	4.59	0.0011	0.02	0.75	12.36	0.03
	Std. Loss in π and $\tilde{u}-$gap, $\lambda=0.0035$					
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(3)	0.34	0.0001	0.22	0.72	11.86	0.32
	Std. Loss in π and $\tilde{u}-$gap, $\lambda=0.0521$					
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(4)	275.93	0.0683	1.96	0.51	8.27	3.83

* Relative to welfare-based optimal commitment, as percent of steady-state consumption.

Table 3: Alternative Policy Objectives						
Discretion						
	Quadratic loss Relative to Opt. Commit. (%)	Welfare-based loss Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(1)	10.50	0.0026	0.39	0.72	11.93	0.54
	Std. Loss in π and $\tilde{c}-$gap, $\lambda = \lambda_0$					
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(2)	4.55	0.0011	0.02	0.75	12.36	0.03
	Std. Loss in π and $\tilde{u}-$gap, $\lambda = 0.0035$					
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(3)	16.75	0.0041	0.45	0.72	12.04	0.62
	Std. Loss in π and $\tilde{u}-$gap, $\lambda = 0.0521$					
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(4)	1936.12	0.4815	4.83	0.43	7.35	1.13

* Relative to welfare-based optimal commitment as percent of steady-state consumption.

Table 4: Alternative Policies						
	Baseline NK model optimal commitment policy					
	Quadratic loss Relative to Opt. Commit. (%)	Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(1)	6.62	0.002	0.04	0.75	12.47	0.05
	Baseline NK model optimal discretionary policy					
	Quadratic loss Relative to Opt. Commit. (%)	Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(2)	6.89	0.002	0.03	0.75	12.48	0.040
	Strict Inflation Targeting					
	Quadratic loss Relative to Opt. Commit. (%)	Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(3)	5.41	0.001	0	0.74	12.41	0
	Taylor Rule: $i_t = (0.5/4)\tilde{c}_t + 1.5\pi_t$					
	Quadratic loss Relative to Opt. Commit. (%)	Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
	1236	0.30	3.89	0.40	6.38	9.72
	Estimated policy rule CGG Volker-Greenspan era: $i_t = 0.71 i_{t-1} + 0.29[(1.72\mathrm{E}_t\tilde{c}_{t+1} + 0.3\mathrm{E}_t\pi_{t+1})$					
	Quadratic loss Relative to Opt. Commit. (%)	Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(4)	8001	1.94	9.71	0.16	2.45	60.7

* Relative to optimal commitment utility level, as percent of steady-state consumption.

Table 5: Alternative Policy Objectives: EU Calibration						
	Welfare-based loss					
	Quadratic loss Relative to Opt. Commit. (%)	Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
Commitment	0	0	0.07	0.47	11.96	0.16
Discretion	0.59	0.0001	0.09	0.47	11.96	0.20
		Std. Loss in π and $\tilde{c}-$gap,$\lambda=\lambda_0$				
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
Commitment	0.70	0.0001	0.01	0.47	12.04	0.01
Discretion	0.66	0.0001	0.01	0.47	12.05	0.02
		Std. Loss in π and $\tilde{u}-$gap, $\lambda=0.0035$				
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
Commitment	0.21	0.0000	0.13	0.47	11.96	0.16
Discretion	2.08	0.0003	0.13	0.47	11.99	0.27
		Std. Loss in π and $\tilde{u}-$gap, $\lambda=0.0521$				
		Welfare cost*	σ_π	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
Commitment	133.62	0.0191	1.02	0.43	10.89	2.38
Discretion	434.18	0.0621	1.76	0.42	11.12	4.15

* Cost of bargaining shocks as percent of steady-state consumption

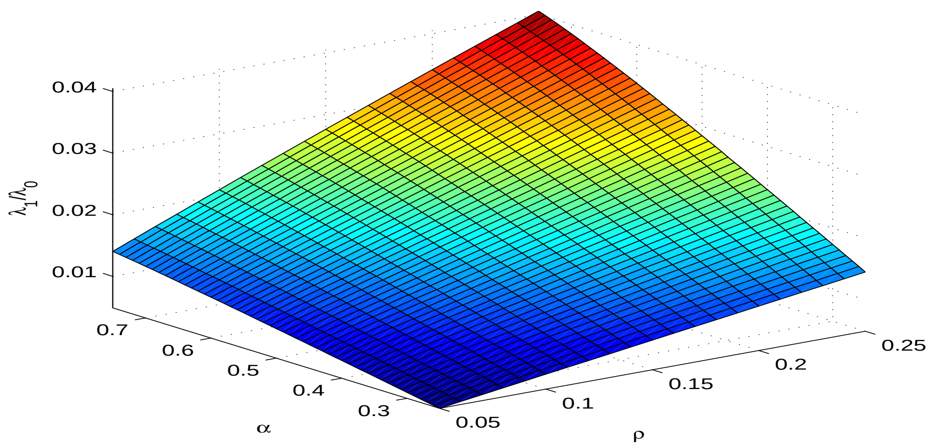

Figure 1: Relative weights on θ and c gaps (λ_1/λ_0) in loss function (29) as functions of α and ρ.

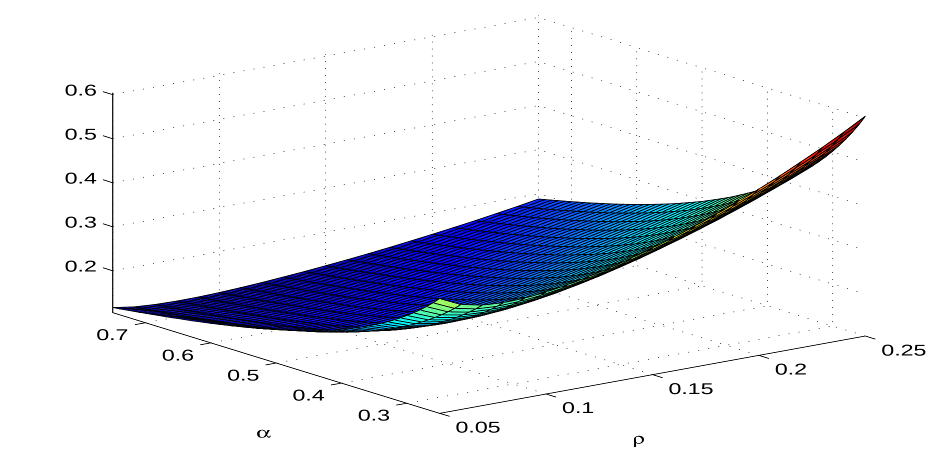

Figure 2: Relative weights on $(\tilde{u}_{t+1} - \rho_u \tilde{u}_t)^2$ and $\tilde{c}_t^2$ ($\bar{\lambda}_1/\lambda_0$) in loss function (30) as functions of α and ρ.

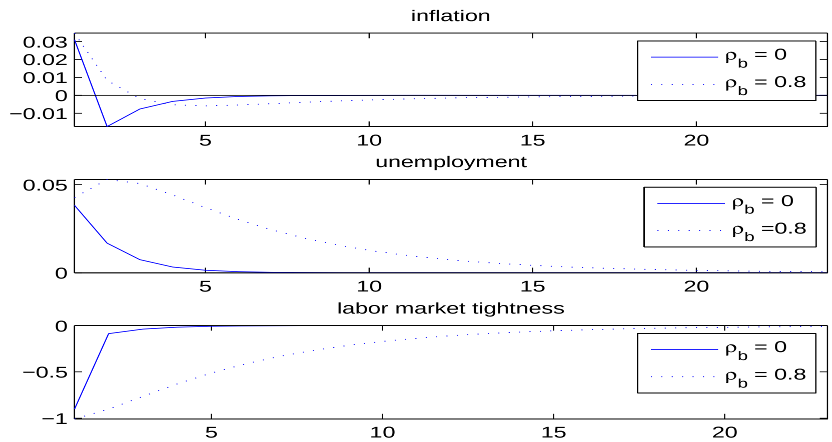

Figure 3: Response to a one standard deviation bargaining shock under optimal commitment for $\rho_b = 0$ and $\rho_b = 0.8$.

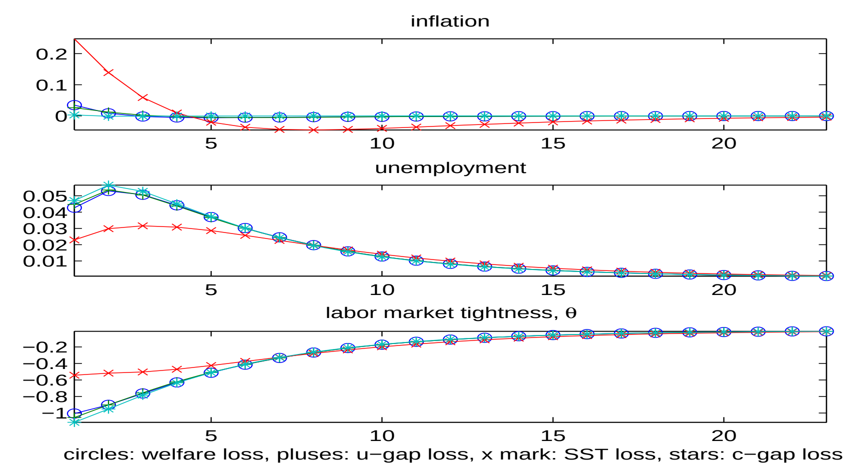

Figure 4: Impulse responses to a one standard deviation bargaining shock under optimal commitment policies minimizing different loss functions

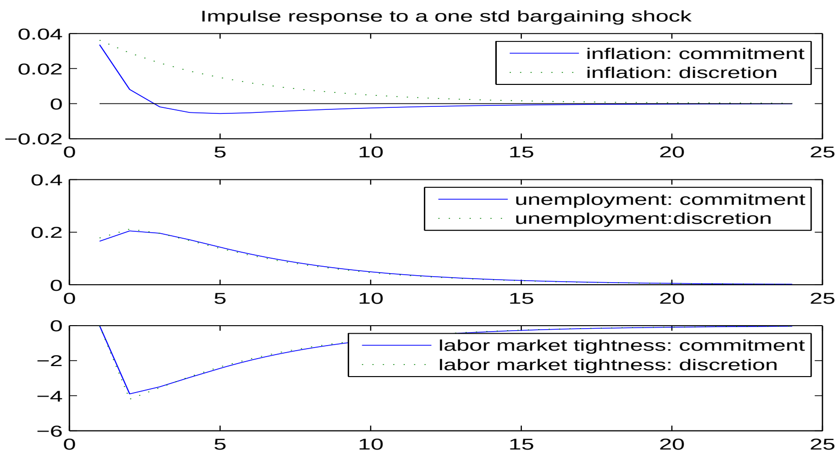

Figure 5: Responses to a one standard deviation bargaining shock under optimal commitment and discretion

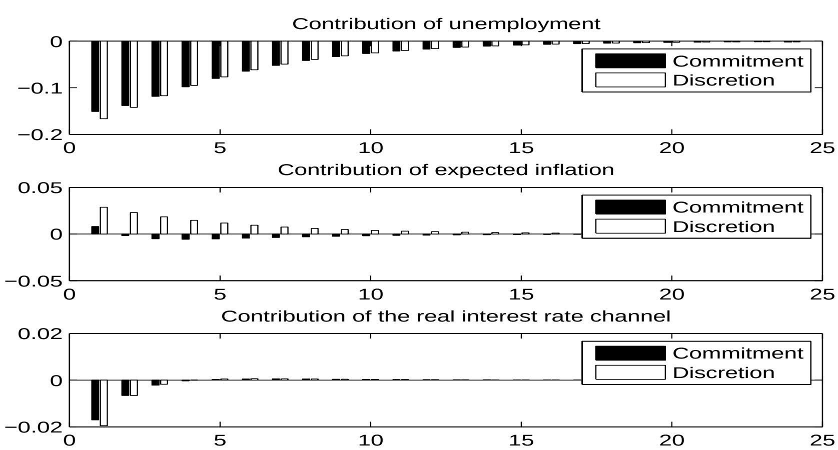

Figure 6: Contribution of labor market, expected inflation, and the real interest rate to the path of inflation net of the direct effect of the bargaining shock when policy is based on the welfare approximation.

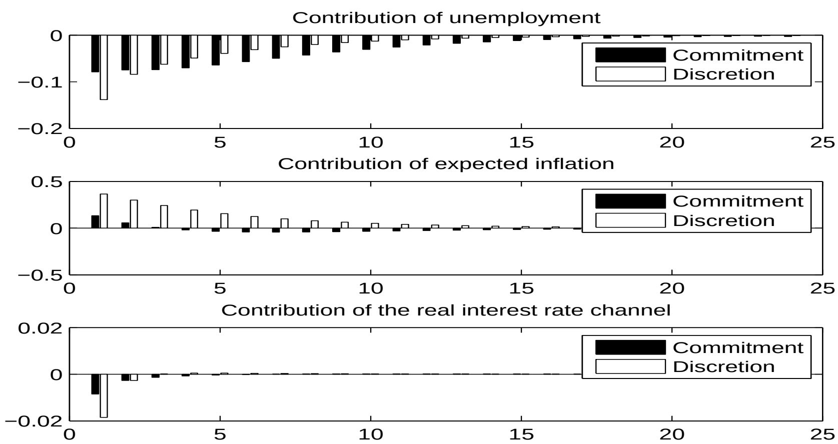

Figure 7: Contribution of labor market, expected inflation, and the real interest rate to the path of inflation net of the direct effect of the bargaining shock when policy is based on the minimizing the exprected present value of $\pi_t^2 + \lambda \tilde{u}_t^2$ and $\lambda = 0.0521$.

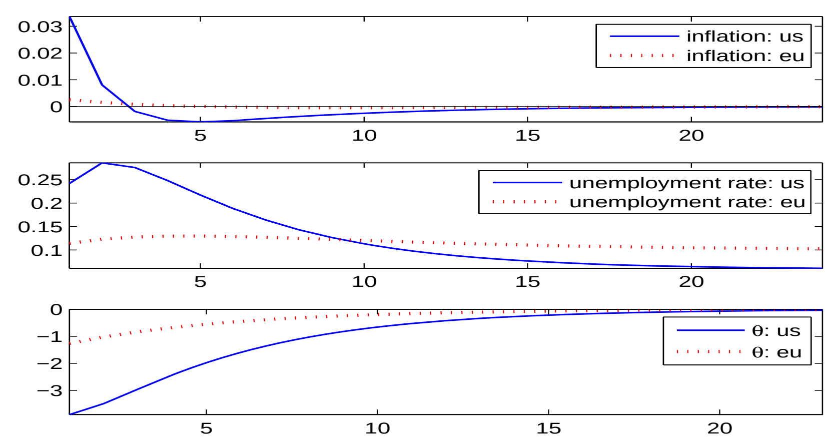

Figure 8: Responses to a one standard deviation bargaining shock for U.S. (solid line) and EU (dotted line) calibrations. (Note: middle panel shows the rate of unemployment.)

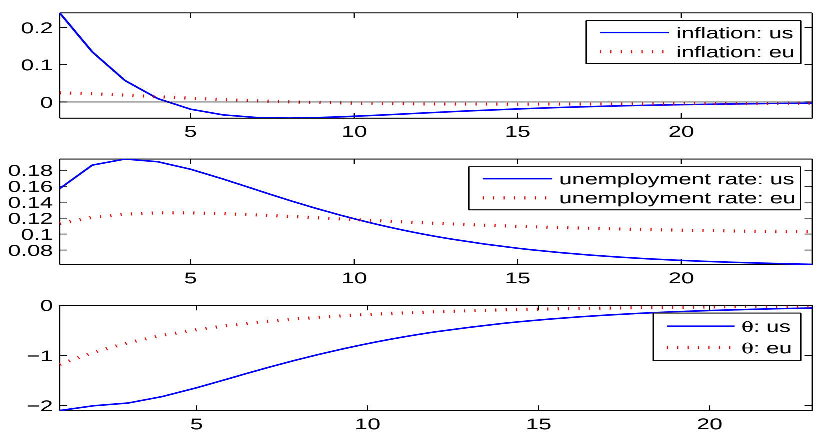

Figure 9: Responses to a one standard deviation bargaining shock for U.S. and EU calibrations when policy minimizes the standard loss function (32) with $\lambda = 0.0521$.