Publication of Interest Rate Forecasts by the Central Bank

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Abstract

This paper studies the role of central bank transparency in the presence of heterogenous information, extending to an infinite horizon the two-period model of Gosselin et al. (2008). Transparency is defined as the publication of interest rate forecasts. Information is heterogenous in the sense that the central bank and the private sector receive different signals about the (single) shock. Transparency results in a mirror effect: the central bank first indirectly reveals its signals when it sets the short term interest rate, and then recovers the private sector's own signals when the latter sets the long term interest rate. Opacity can welfare dominates transparency when forecast errors of the central bank and the private sector offset each other.

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1 Introduction

The practice of producing and publishing policy interest rate forecasts adopted by some central banks -the Reserve Bank of New Zealand, the Bank of Norway, the Central Bank of Iceland and the Swedish Riksbank- is controversial. Supporters argue that this is the only way of producing consistent forecasts of output and inflations, as many central banks currently do. Opponents respond that the practice of using market forecasts of interest rates -extracted for instance from observable yield curves- is equally consistent, at least as long as the central bank agrees with the market forecasts. This supposes, however, that the inflation and output forecasts are those that the central bank wishes to see over the forecast horizon, otherwise it must be the case that the central bank will take action to try to correct any divergence from its most desired outcome. Put differently, using market forecasts to produce output and inflation forecasts is consistent only if the markets correctly predict future monetary policy actions. Thus, whether it publishes its own policy interest rate forecast or not, for forecasting consistency a central bank must make its intentions perfectly transparent to the market.

Forecasting consistency is not the only consideration driving the communication strategy, however. Many central banks argue that they need to retain some room for maneuver to react to unexpected events or even changing views within the policy-making committee. Publishing the interest rate path, they claim, would be interpreted as a commitment that would be hard to deviate from, even if new events or new analysis would warrant doing so. This argument conflates two issues. One is time consistency and whether central banks should react to news. The answer, as shown in Woodford (2003) is for the central bank to specify -explicitly or implicitly- a decision rule. The other issue is whether actions based of new information may be misconstrued as time inconsistency. This is a delicate question which involves the relative information sets of the central bank and of the private sector and how these two players exchange, directly or indirectly, information.

The two questions -the publication of the expected interest rate path and misconstrued time inconsistency- are deeply related since they involved the exchange of information sets between the central bank and the private sector. Following Morris and Shin (2002), a small but growing literature has started to explore this issue. Faust and Leeper (2005) assume that the central bank holds an information advantage over the private sector, which in their model implies that sharing that information is welfare-enhancing. It follows that conditional forecasts - i.e. not revealing the policy interest path - provide less useful information than unconditional forecasts, for which they find some supporting empirical evidence. Rudebusch and Williams (2006) assume an information asymmetry between the central bank and the private sector regarding both policy preferences and targets. Allowing for a "transmission noise" that distorts central bank communication, they find that revealing the expected path improves the estimation process and welfare, with a gain that declines as the transmission noise increases.[1] Walsh (2007) also allows for information asymmetries and finds that the publication by the central bank of its output gap forecasts - which is equivalent in his model to revealing expected inflation - can have a welfare-reducing effect if the central bank is poorly informed. In the spirit of Morris and Shin (2002), he concluded that the optimal degree of transparency depends on the relative accuracy of central bank information.

Gosselin et al. (2008) take a different route by assuming instead an information *heterogeneity* between the central bank and the private sector. They find that publishing the interest rate path leads to an exchange of information: the private sector observes the interest rate set by the central bank and the central bank observes financial prices. When neither side can ever fully recover the information of the other side by simply observing its actions, it is possible for opacity to welfare-dominate transparency. The present paper offers an extension of that two-period model to an indefinite horizon. Most of the previous results are preserved. The richer environment provides a richer description of the influence of transparency and of the effect of the associated intertemporal exchange of information on the yield curve. The richer model also enhances the case for transparency because it provides the central bank with private sector forecasts of the future, which is useful over the extended horizon.

2 The Model

2.1 The Economy

As in Gosselin et al. (2008), we describe the economy with the standard New-Keynesian log-linear model. It includes a Phillips curve:

$$\pi_t = RE_t^P \pi_{t+1} + \kappa_1 y_t + \varepsilon_t \tag{1}$$

where y_t is the output gap and ε_t is a random disturbance uniformly distributed over the real line, with an improper distribution and a zero unconditional mean. Without loss of generality, we can assume in what follows a zero rate of time preference so that $R = 1$. The output gap is given by the forward-looking IS curve:

$$y_t = E_t^P y_{t+1} - \kappa_2 (r_t - E_t^P \pi_{t+1} - r^*) \tag{2}$$

where r_t is the nominal interest rate. Note that we do not allow for a demand disturbance to keep the model simple and focus on the role of information. We further assume that the natural real interest rate $r^* = 0$.

In both equations, the expectations E^P are those of the private sector since prices and output are private decisions, which we assume are made after the central bank

[1] They examine the role of central bank signal accuracy and find that its underestimation by the private sector limits the gains from releasing the expected interest path while overestimation may be counterproductive.

has decided on the contemporaneous interest rate. The reason for this assumption is the usual one: prices are sticky, for example because eq. (1) is based on Calvo pricing by a large number of producers so that, at any moment, only a small number of prices are reset while the central bank makes decisions at a high frequency. We consider an infinite number of discrete periods ranging from $-\infty$ to $+\infty$, in line with the timeless perspective solution to the time inconsistency problem. It follows that steady states are identical across time.

2.2 The Loss Function

The central bank minimizes an intertemporal loss function, which also assumes that the gross rate of time preference R is unity. In addition, the period loss only involves deviations of actual from target inflation. We do not allow for the output gap to affect welfare. This assumption eliminates the standard inflation-output trade-off. It also eliminates the possibility that the public may be uncertain about central bank preferences. Both issues are overlooked here because they are well-known and have already been studied, while adding complexity to an already rich set of issues. Further setting the target inflation rate to zero, the loss function can be written as the following unconditional expectation:

$$L_t = E \sum_{n=0}^{\infty} \pi_{t+n}^2 \tag{3}$$

A closed form for the inflation π_t, which involves an infinite series in the expectations of the future variables, will be given only at a later stage. Let us first turn to the information set-up which will allow to impose some restrictive conditions for the formation of expectations.

2.3 Information Structure and timing

For information to matter at all, it must be that the central bank and the private sector have different information sets. Much of the literature on central bank transparency assumes information asymmetry, holding that the central bank knows more things -including its own preferences- than the private sector. We take a different approach, focusing instead on information heterogeneity, which arises because the central bank and the private sector receive different signals about the shock ε_t.

In order to meaningfully discuss forecasts of future interest rates, the central bank must use currently available information about future shocks. Naturally, the private sector too will use its own currently available information about future shocks to make its own forecasts and to interpret the central bank forecasts when they are published. We thus need both the central bank and the private sector agents to receive information in advance, before the realization of the shocks.

We implement this idea by assuming that information about ε_t becomes available in period $t-1$ and that it is updated before decisions are made in period t. Both the early signal $\varepsilon^j_{t-1,t}$ and its update $\hat{\varepsilon}^j_{t,t}$ are centered around ε_t. These signals must be carefully distinguished from the forecasts $E^j_{t-1}\varepsilon_t$ and $E^j_t\varepsilon_t$ formed in periods $t-1$ and t, respectively by agent j, where $j = CB, P$ denotes the recipient of the signals, the central bank and the private sector.

The notation difference between $\varepsilon^j_{t-1,t}$ and $\hat{\varepsilon}^j_{t,t}$ has a practical reason. At time t, each agent has received three signals: one signal about the future shock $\varepsilon^j_{t,t+1}$ and two signals about the contemporaneous shock $\hat{\varepsilon}^j_{t,t}$ and $\varepsilon^j_{t-1,t}$. Using Bayes rule, it is always possible to combine $\hat{\varepsilon}^j_{t,t}$ and $\varepsilon^j_{t-1,t}$ to form an improved signal $\varepsilon^j_{t,t}$, which is the only one needed to form all other expectations. Put differently, given $\varepsilon^j_{t-1,t}$, $\varepsilon^j_{t,t}$ supersedes $\hat{\varepsilon}^j_{t,t}$.

The variances of all signals are constant and public knowledge. Early signals $\varepsilon^j_{t-1,t}$ have known variances $(k\alpha)^{-1}$ and $(k\beta)^{-1}$ for the central bank and the private sector, respectively. Equivalently, the signal precisions are $k\alpha$ and $k\beta$. Similarly, the updated signals $\hat{\varepsilon}^j_{t,t}$ have variances $[(1-k)\alpha]^{-1}$ and $[(1-k)\beta]^{-1}$ for the central bank and the private sector respectively. This implies that the recombined signals $\varepsilon^j_{t,t}$ have variance α^{-1} and β^{-1} i.e. their respective precisions are α and β. Thus parameter k measures the relative prevision of early signals vis a vis the updated signals. We assume that $0 \leq k \leq 1.$, i.e. that early signals are less precise than updated signals. We will also define the relative precision of central bank signals $z = \alpha/\beta$, with no restriction on z except that it is non-negative.

To complete the description of the information structure, we assume that, at the beginning of period t, the realized values of π_{t-1} and ε_{t-1} become known to both the central bank and the private sector. The central bank uses all information available - the early and contemporaneous signals $\varepsilon^{CB}_{t,t+1}$ and $\varepsilon^{CB}_{t,t}$ as well as π_{t-1} and ε_{t-1} - to form its forecast $E^{CB}_t\varepsilon_t$. The central bank then uses all available information to decide on the interest rate r_t that minimizes $E^{CB}_t L_t$. After the central bank decision, the private sector observes r_t, forms its expectations and decides on output and prices.

2.4 Inflation dynamics

Since at time t the central bank and the private sector have no information on shocks beyond period $t+1$, their best estimates are the unconditional means $E\varepsilon_{t+i} = 0$, for $i > 1$. As we will see, this does not imply that $E^j_t\pi_{t+i}$ and $E^j_t y_{t+i}$ are nul for $i > 1$. The reason is that each agent inevitably makes forecast errors and the other knows it.

We now expand (1) and (2) as series of expectations of the shocks and the future interest rates. To do so,we write the dynamical system (1), (2) matricially:

$$E^P_t A \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} \varepsilon_t \\ -\kappa_2 r_t \end{pmatrix}$$

where $A = \begin{pmatrix} 1-L & -\kappa_1 \\ -\kappa_2 L & 1-L \end{pmatrix}$ and L is the lead operator. The solution of the system is given by:

$$\begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = E_t^P A^{-1} \begin{pmatrix} \varepsilon_t \\ -\kappa_2 r_t \end{pmatrix}$$

Defining $\kappa = \kappa_1 \kappa_2$ and concentrating on inflation dynamics, the last equation can be expanded in terms of the lead operator:

$$\pi_t = E_t^P \frac{1-L}{1-2L+L^2-\kappa L}\varepsilon_t - E_t^P \frac{\kappa}{1-2L+L^2-\kappa L} r_t$$

or, separating the contributions of the shocks and the interest rate:

$$\pi_t = \varepsilon_t + (1+\kappa) E_t^P \varepsilon_{t+1} - \frac{\kappa}{1-2L+L^2-\kappa L} r_t$$

Then to find a series expansion for π_t, we use the standard decomposition for the quadratic operator in L:

$$1 - (2+\kappa) L + L^2 = (1-\varsigma L)\left(1-\varsigma^{-1} L\right)$$

with $\varsigma = \left(1 + \frac{1}{2}\kappa - \frac{1}{2}\sqrt{(4\kappa+\kappa^2)}\right)$. This leads ultimately to:

$$\begin{aligned} \pi_t &= \varepsilon_t + (1+\kappa) E_t^P \varepsilon_{t+1} - \kappa E_t^P \sum_{n=0}^{\infty} \sum_{m=0}^{n} \varsigma^{2m-n} r_{t+n} \\ &= \varepsilon_t + (1+\kappa) E_t^P \varepsilon_{t+1} - \kappa E_t^P \sum_{n=0}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} r_{t+n} \\ &= \varepsilon_t + (1+\kappa) E_t^P \varepsilon_{t+1} - \kappa r_t - \kappa (2+\kappa) E_t^P r_{t+1} - \kappa E_t^P \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} r_{t+n} \end{aligned} \quad (4)$$

Since it is forward-looking, current inflation depends on current and expected shocks and on a current and expected policy interest rates. This implies that the central bank must take into account the effect of its current and future decisions on market expectations. Put differently, the central bank must forecast how private sector forecasts will react to the choice of r_t. Shocks beyond period $t+1$ do not appear in (4) because there is no available information about them so the market best forecast is the unconditional mean, which is zero. It may be suprising that all future interest rates matter. After all, with $E_t^P \varepsilon_{t+i} = 0$ for all $i \geq 2$, the private sector should have no reason to expect any policy action. As will become clear below, this is not generally the case because (1) and (2) imply that forecasting errors on inflation and the output gap are autoregressive.

2.5 Signal extraction

In the following sections, we will study various equilibria corresponding to central bank decisions based on information available at decision time. At time t, the central bank has received signals $\varepsilon_{t,t}^{CB}$ and $\varepsilon_{t,t+1}^{CB}$ and it also knows the realized values of π_{t-1} and ε_{t-1}. As a consequence, from (4) the central bank also knows:

$$\pi_{t-1} - \varepsilon_{t-1} + \kappa r_{t-1} = (1+\kappa) E_{t-1}^{P} \varepsilon_t - \kappa (2+\kappa) E_{t-1}^{P} r_t - \kappa E_{t-1}^{P} \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} r_{t-1+n} \tag{5}$$

This shows that the publication of statistical data, presumed here to be precise, provides the central bank with some information about private sector's previous period expectations. The information comes as a linear combination of forecasts including future policy interest rates. This means that the central bank can only extract from period $t-1$ data only one signal and that the precise form of this signal will depend on the communication regime. Still, the following remarks hold independently of the regime.

Since the new information is dated $t-1$, the signal, denoted s_t, is only about ε_t, and it can always be centered around ε_t. In addition the right-hand side of (5) involves expectations based on private sector signals $\varepsilon_{t-1,t-1}^{P}$ and $\varepsilon_{t-1,t}^{P}$. It follows that we can postulate that s_t is a linear combination of $\varepsilon_{t-1,t-1}^{P}$ and $\varepsilon_{t-1,t}^{P}$, say $\varepsilon_{t-1,t}^{P} - \theta \varepsilon_{t-1,t-1}^{P}$ after normalizing the coefficient of $\varepsilon_{t-1,t}^{P}$ to unity. In order to obtain a signal s_t centered around ε_t, the central bank will need to cancel the mean contribution of $\varepsilon_{t-1,t-1}^{P}$. To that effect, s_t must be of the general form:

$$s_t = \varepsilon_{t-1,t}^{P} - \theta \left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1} \right) \tag{6}$$

where $\tilde{\varepsilon}_{t-1}$ is chosen by the central bank to minimize the variance of the noisy contribution $\left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1} \right)$. We will see that θ is a parameter that depends on the communication regime.

As it tries to disentangle $\varepsilon_{t-1,t-1}^{P}$ from $\varepsilon_{t-1,t}^{P}$ to understand the private signals received in period $t-1$ about ε_t, the central bank must correct as best as it can the error included in $\varepsilon_{t-1,t-1}^{P}$. This introduces some autoregressivity. Indeed, we show in the Appendix that the last term in (6) satistifies the following condition:

$$\begin{aligned} \left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1} \right) &= ((1-k)+kx) \left(\varepsilon_{t-1,t-1}^{P} - \varepsilon_{t-1} \right) + k(1-x) \left(\varepsilon_{t-1,t-1}^{P} - \varepsilon_{t-2,t-1}^{P} \right) \\ &\quad + k(1-x)\,\theta \left(\varepsilon_{t-2,t-2}^{P} - \tilde{\varepsilon}_{t-2} \right) \end{aligned}$$

where the coefficient x is given by :

$$x = \frac{1}{2k^2\theta^2} \left(-1 - k\theta^2 + 2k^2\theta^2 + \sqrt{\left(1 + 2k\theta^2 - 4k^2\theta^2 + k^2\theta^4\right)} \right)$$

The Appendix also establishes that $\tilde{\varepsilon}_{t-1}$ is known to the private sector at time t.

Thus the signal s_t recovered from the publication of π_{t-1} and ε_{t-1} in period t can be interpreted by the central bank as a linear combination of private sector signals made in the previous period. The weights involve parameter θ, which depends on the way the private sector forms its expectations of future interest rates. This parameter will depend therefore on the information released by the central bank. Note, for instance, that $x \to 0$ when $\theta \to 0$, so that $s_t = \varepsilon^P_{t-1,t}$: in this case the central bank can fully recover, period after period, early private sector signals. We will see that this occurs when the central bank is fully transparent.

2.6 Optimal policy[2]

Finally, we determine the optimal decision of the central bank. As in Gosselin et al. (2008), in order to find analytical solutions, we restrict our analysis to Taylor rules of the form :

$$r_t = \mu \varepsilon^{CB}_{t,t} + \nu \varepsilon^{CB}_{t,t+1} + \xi s_t \tag{7}$$

The central bank needs to find the optimal values of μ, ν and ξ. Here we establish a constraint on this choice. The inflation dynamics equation (4) involves private sector expectations of r_{t+n}. In period t there is no information on shocks beyond $t+1$ so (7) implies that $E^P_t r_{t+n} = \xi E^P_t s_{t+n}$ for $n \geq 2$. We show that there exists a parameter x such that $\tilde{\varepsilon}_t = ((1-k) + kx)\, \varepsilon_t + k(1-x)\, s_t$. Along with (6), this implies:

$$s_t = \varepsilon^P_{t-1,t} - \theta \left(\varepsilon^P_{t-1,t-1} - [((1-k) + kx)\, \varepsilon_{t-1}] \right) + \theta k (1-x)\, s_{t-1} \tag{8}$$

This last relation can be iterated forward to find the infinite sum in (4). Actually, given (7), one has

$$-\kappa E^P_t \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} r_{t+n} = -\kappa \xi \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} E^P_t s_{t+n} \tag{9}$$

and a computation shows that:

$$-\kappa \xi \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} E^P_t s_{t+n} = \tau \left[\left(k\varepsilon^P_{t,t+1} - kx E^P_t \varepsilon_{t+1} \right) - k(1-x)\, E^P_t s_{t+1} \right]$$

with:

$$\tau = \frac{\theta \kappa \xi}{(1-\varsigma^2)} \left(\frac{1}{\varsigma^2} \frac{1}{1 - \frac{\theta k (1-x)}{\varsigma}} - \frac{\varsigma^2}{1 - \varsigma \theta k (1-x)} \right)$$

As a consequence 4 can be rewritten as:

$$\begin{aligned} \pi_t \quad = \quad & -(2+\kappa)\, \kappa E^P_t r_{t+1} - \kappa r_t + \varepsilon_t + (1+\kappa)\, E^P_t \varepsilon_{t+1} \\ & + \tau \left(k \varepsilon^P_{t,t+1} - kx E^P_t \varepsilon_{t+1} - k(1-x)\, E^P_t s_{t+1} \right) \end{aligned} \tag{10}$$

[2]Computation details can be found in the Appendix.

Substituting (9) into (4), we ultimately obtain the following expression for the inflation:

$$\begin{aligned}\pi_t &= -\left(2+\kappa\right)\kappa E_t^P\left(\mu\varepsilon_{t+1,t+1}^{CB}+\xi s_{t+1}\right)-\kappa\left(\nu\varepsilon_{t,t+1}^{CB}\right)+\left(1+\kappa\right)E_t^P\varepsilon_{t+1} \\ &\quad+\left(\varepsilon_t-\kappa\left(\mu\varepsilon_{t,t}^{CB}+\xi s_t\right)\right)+\tau\left(k\varepsilon_{t,t+1}^P-kxE_t^P\varepsilon_{t+1}-k\left(1-x\right)E_t^P s_{t+1}\right)\end{aligned} \quad (11)$$

which shows how inflation relates to the shocks, to the signals received by the central bank and the private sector and to the parameters of the Taylor rule that the central bank chooses in order to minimize (3). Optimal policy therefore requires to minimize the variance of the above expression. Since the shocks ε_t are assumed to be uniformally distributed, their variance is infinite. The signals are centered around those shocks so they can be thought of as: $\varepsilon_{t,t}^{CB}=\varepsilon_t+$ noise, $\varepsilon_{t,t+1}^{CB}=\varepsilon_{t+1}+$ noise, etc., with noises of finite variance because signal precision is positive. The same is true for composed expectations such as $E_t^P E_{t+1}^{CB}\varepsilon_{t+1}=\varepsilon_{t+1}+$ noise and of the signal $s_t=\varepsilon_t+$ noise. It follows that $\tau\left(k\varepsilon_{t,t+1}^P-kxE_t^P\varepsilon_{t+1}-k\left(1-x\right)s_{t+1}\right)=0+$terms of finite variances and we can rewrite the inflation equation as :

$$\pi_t=\left[-\left(2+\kappa\right)\kappa\left(\mu+\xi\right)-\kappa\nu+\left(1+\kappa\right)\right]\varepsilon_{t+1}+\left[1-\kappa\left(\mu+\xi\right)\right]\varepsilon_t+\text{ terms of finite variances}$$

As it chooses μ, ν and ξ to minimize (3), the central bank cancels the coefficients of ε_t and ε_{t+1} since these terms are of infinite variance:

$$\begin{aligned}\left(-\left(2+\kappa\right)\kappa\left(\mu+\xi\right)-\kappa\nu+\left(1+\kappa\right)\right) &= 0 \\ \left(1-\kappa\left(\mu+\xi\right)\right) &= 0\end{aligned}$$

It follows that $\mu+\xi=\frac{1}{\kappa}$ and $\nu=-\frac{1}{\kappa}$. We define ρ such that $\mu=\frac{\rho}{\kappa}$, so we have $\xi=\frac{1-\rho}{\kappa}$ and the Taylor rule is restricted to:

$$r_t=\frac{1}{\kappa}\left[\rho\varepsilon_{t,t}^{CB}+\left(1-\rho\right)s_t-E_t^{CB}\varepsilon_{t+1}\right] \quad (12)$$

The two restrictions on the policy parameters mean that optimal policy boils down to the choice of a single parameter, ρ.

3 The Central Bank Reveals its Interest Rate Forecast

We first look at the case where the central banks reveals the expected policy interest rate path. Since the central bank has no private information beyond $t+1$, the only forecast of interest is $E_t^{CB}r_{t+1}$. This conforms to the practive of central banks that publish their interest rate forecasts up to a horizon of two or three years. We refer to this communication strategy as transparency.

When it announces $E_t^{CB} r_{t+1}$ alongside the current policy decision r_t, the central bank in fact releases two independent signals $\varepsilon_{t,t}^{CB}$ and $\varepsilon_{t,t+1}^{CB}$. To see that, consider the private sector's forecasting exercise. The Taylor rule (12) and knowledge of $E_t^{CB} r_{t+1}$ implies that the private sector observes a linear combination of $E_t^{CB} \varepsilon_{t+1,t+1}^{CB} = \varepsilon_{t,t+1}^{CB}$, $E_t^{CB} s_{t+1}$ and $E_t^{CB} \varepsilon_{t+2}$.Note that $E_t^{CB} \varepsilon_{t+2} = 0$, given our assumed truncation of signals. Next, (8) implies that:

$$\begin{aligned} E_t^{CB} s_{t+1} &= E_t^{CB} \left[\varepsilon_{t,t+1}^{P} - \theta \left(\varepsilon_{t,t}^{P} - \left[\left((1-k) + kx \right) \varepsilon_t \right] \right) + \theta k (1-x) s_t \right] \\ &= \varepsilon_{t,t+1}^{CB} - \left[\theta \left(E_t^{CB} \varepsilon_{t,t}^{P} - \left[\left((1-k) + kx \right) E_t^{CB} \varepsilon_t \right] \right) \right] + \theta k (1-x) s_t \end{aligned}$$

Since $E_t^{CB} \varepsilon_{t,t}^{P}$ and $E_t^{CB} \varepsilon_t$ both combine $\varepsilon_{t,t}^{CB}$ and s_t, $E_t^{CB} r_{t+1}$ is a combination of $\varepsilon_{t,t+1}^{CB}$, $\varepsilon_{t,t}^{CB}$ and s_t is a signal known to the private sector. It follows that the observation of r_t and $E_t^{CB} r_{t+1}$ provides the private sector with two linear combinations of $\varepsilon_{t,t}^{CB}$ and $\varepsilon_{t,t+1}^{CB}$. As a consequence, the private sector can infer both $\varepsilon_{t,t}^{CB}$ and $\varepsilon_{t,t+1}^{CB}$.

This makes good intuitive sense. The optimal choice of the current and future interest rates requires that the central bank uses all available information about current and future disturbances. In this model, where there is just one disturbance per period and where the length of a period is chosen such that there is no information beyond $t+1$, the central bank has two private signals and transparency about two interest rates makes the central bank fully transparent. Extending the horizon further would not change the result but adding more shocks per period would imply that the central bank does not reveal all its private information, only two linear combinations of its signals.

With two signals about the future shock, the private sector uses Bayes rule to make the following forecast:

$$E_t^{P} \varepsilon_{t+1} = \gamma_1^{tr} \varepsilon_{t,t+1}^{P} + (1-\gamma_1^{tr}) \varepsilon_{t,t+1}^{CB} = \frac{1}{1+z} \varepsilon_{t,t+1}^{P} + \frac{z}{1+z} \varepsilon_{t,t+1}^{CB} \tag{13}$$

where $z = \alpha/\beta$ is the precision of central bank signals relative to those of the private sector.

Under transparency, we show in the Appendix that the parameter θ in (6) is equal to zero so that $s_t = \varepsilon_{t-1,t}^{P}$. Thus, when in period t the realized values of π_{t-1} and ε_{t-1} become known, the combined signal s_t allows the central bank to recover the private sector signal $\varepsilon_{t-1,t}^{P}$. The reason is that transparency leads the private sector to forecast $E_{t-1}^{P} \varepsilon_t$ with Bayes rule by combining $\varepsilon_{t-1,t}^{P}$ and $\varepsilon_{t-1,t}^{CB}$, ignoring its own early signal $\varepsilon_{t-1,t-1}^{P}$. Transparency provides the central bank with the opportunity to observe how its own signal $\varepsilon_{t-1,t}^{CB}$ has been interpreted by the private sector.

Proposition 1 *Delayed mirror effect. When it announces $E_t^{CB} r_{t+1}$ in period t, the central bank indirectly reveals $\varepsilon_{t,t+1}^{CB}$, its signal about ε_{t+1}. Then, in the next period, the central bank receives the signal $s_{t+1} = \varepsilon_{t,t+1}^{P}$, which allows it to indirectly observe the private sector signal about ε_{t+1}. As a consequence, the forecasts of the central bank* *and of the private sector are prefectly aligned under transparency and the private sector expects future inflation to be zero.*

Before proving the last part of the proposition, we note that the fact that $\varepsilon^P_{t-1,t-1}$ is ignored by the private sector in period $t-1$, and by the central bank when it sets r_t and $E^{CB}_t r_{t+1}$ in period t, means that the corresponding past forecast error is inconsequential. This eliminates the autoregressivity of the signal s_t and inflation does not depend on future interest rates beyond the next period. Indeed, since $\tau = 0$ when $\theta = 0$, from (9) we see that $-\kappa \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n(1-\varsigma^2)} E^P_t r_{t+n} = 0$ so that (4) reduces to:

$$\pi_t = -(2+\kappa)\,\kappa E^P_t r_{t+1} - \kappa r_t + \varepsilon_t + (1+\kappa)\, E^P_t \varepsilon_{t+1} \tag{14}$$

Put differently, the far end of the yield curve - which central banls can hardly influence in practice - does not provide any relevant information.

In period $t+1$, the central bank receives three signals about ε_{t+1}: $\varepsilon^{CB}_{t,t+1}$ received in period t with precision $k\alpha$, $\hat{\varepsilon}^{CB}_{t+1,t+1}$ received in period $t+1$ with precision $(1-k)\alpha$ and now $s_{t+1} = \varepsilon^P_{t,t+1}$ of known precision $k\beta$. As previously explained, the central bank combines the two first signals to form $\varepsilon^{CB}_{t+1,t+1}$ with precision α. Applying Bayes rule, we have:

$$E^{CB}_{t+1}\varepsilon_{t+1} = \frac{z}{k+z}\varepsilon^{CB}_{t+1,t+1} + \frac{k}{k+z}\varepsilon^P_{t,t+1}$$

Using the definition of $\varepsilon^{CB}_{t+1,t+1}$, we find:

$$E^P_t E^{CB}_{t+1}\varepsilon_{t+1} = \frac{1}{1+z}\varepsilon^P_{t,t+1} + \frac{z}{1+z}\varepsilon^{CB}_{t,t+1} = E^P_t \varepsilon_{t+1}$$

Knowing that its signal s_t will be taken into account in $t+1$ by the central bank when it optimally decides on r_{t+1}, the private sector is reassured that its expectations are aligned with those of the central bank.

In addition, since inflation only depends linearly on the current interest rate, not on past interest rates, the central bank can always choose r_t to minimize $E^{CB}_t \sum_{i=0}^{\infty} L_{t+i}$ which implies setting $E^{CB}_t \pi_t = 0$. Using (14) and noting that $E^{CB}_t E^p_t = E^{CB}_t$ because the private sector has recovered the central bank signals, we find:

$$r_t = \frac{1}{\kappa}\left[-(2+\kappa)\,\kappa E^{CB}_t r_{t+1} + (1+\kappa)\, E^{CB}_t \varepsilon_{t+1}\right] + \varepsilon_t$$

Next, since $s_t = \varepsilon^P_{t-1,t}$ (12) implies:

$$\kappa E^{CB}_t r_{t+1} = \varepsilon^{CB}_{t,t+1} = E^{CB}_t \varepsilon_{t+1}$$

Together, these two relationships show that under transparency the central bank sets the interest rate in a simple and intuitive way:

$$r_t = \frac{1}{\kappa}\left(E_t^{CB}\varepsilon_t - E_t^{CB}\varepsilon_{t+1}\right)$$

The central bank raises the policy interest rate to offset an expected contemporary inflationary shock ε_t and lowers it if it expects a future inflation shock which steepens the yield curve and raises the long-term interest rate.

It is immediately checked that, with this policy rule, (14) implies that:

$$E_t^P \pi_{t+1} = E_t^P(\varepsilon_{t+1} - E_{t+1}^{CB}\varepsilon_{t+1}) = E_t^P\varepsilon_{t+1} - E_t^P E_{t+1}^{CB}\varepsilon_{t+1} = 0$$

The private sector fully trusts the transparent central bank to deliver the optimal inflation rate, which is zero under the model's assumption. The actual inflation rate, though, will not be zero. Given that $E_t^P E_{t+1}^{CB} = E_t^P$, we have $E_t^P r_{t+1} = \frac{1}{\kappa}\left(E_t^P\varepsilon_{t+1} - E_t^P\varepsilon_{t+2}\right) = \frac{1}{\kappa}E_t^P\varepsilon_{t+1}$, which implies:

$$\pi_t = (\varepsilon_t - E_t^{CB}\varepsilon_t) + (E_t^{CB}\varepsilon_{t+1} - E_t^P\varepsilon_{t+1}) \tag{15}$$

Thus, were it not for two unavoidable forecast errors, actual inflation would be nil, as is optimal. If the central bank underestimates the shock ε_t, it will set the interest rate too low. Forecasts about the future shock also matter because they affect the expected interest rate and the yield curve. For instance, consider the case when the private sector expects a larger shock ε_{t+1} than the central bank. The central bank knows that $E_t^P\pi_{t+1} = 0$ so its choice of r_t is driven by current conditions. Since $E_t^P\varepsilon_{t+1} = E_t^P E_{t+1}^{CB}\varepsilon_{t+1}$, the private sector is not aware of the discrepancy. As a result, the private sector steepens the yield curve - the one-period ahead market interest rate $E_t^P r_{t+1}$ is high - which depresses demand and inflation.

Finally, the welfare loss is:

$$\beta L_t^{\text{tranparency}} = \left(\beta E\pi_t^2\right)^{\text{tranparency}} = \frac{1}{kz\,(1+z)} + \frac{1}{z+k}$$

The loss is normalized using the precision β of private sector signals.

4 The Central Bank Does not Reveal its Interest Rate Forecast

In the opacity regime the central bank only reveals the current interest rate. The policy decision is guided by information available to the central bank: $\varepsilon_{t,t}^{CB}$, $\varepsilon_{t,t+1}^{CB}$ and the complex signal s_t that is revealed when π_{t-1} and ε_{t-1} become known. In constrast to the transparency case, there is no reason now for $\theta = 0$ so the central bank is not able to recover from s_t the private signal $\varepsilon_{t-1,t}^P$. This is because, in period $t-1$, the private sector forms $E^P_{t-1}\varepsilon_t$ by combining the information available from the policy rate r_{t-1} with its own signal about the present shock, $\varepsilon^P_{t-1,t-1}$. It follows that s_t mixes up $\varepsilon^P_{t-1,t}$, which is useful for the period t decision, with $\varepsilon^P_{t-1,t-1}$, which has no relevant information content. Not only is s_t noisy, but it is also autocorrelated. The main result is summarized as follows:

Proposition 2 *Private and central bank expectations are no longer aligned under opacity. While the interest rate rule is the same - but the coefficients different - as when the central bank announces its expected future interest rate, the yield curve no longer matches the central bank forecast of the interest rate path.*

To establish these results, we first need to list the various agents expectations of the present and future shocks. The coefficients being computed later by identification[3]. Starting with the Central Bank ones, a direct application of the Bayes rule yields:

Looking first at the central bank forecasts, Bayes rule implies the following:

$$E^{CB}_t\varepsilon_t = \gamma_4\varepsilon^{CB}_{t,t} + (1-\gamma_4)\,s_t$$

$$E^{CB}_t\varepsilon^P_{t-1,t} = \gamma_0\varepsilon^{CB}_{t,t} + (1-\gamma_0)\,s_t$$

$$E^{CB}_t\varepsilon^P_{t,t} = kE^{CB}_t\varepsilon^P_{t-1,t} + (1-k)\,E^{CB}_t\varepsilon_t$$

with :

$$\begin{aligned}\gamma_4 &= \frac{z\,(1+kf\,(\theta))}{k+z\,(1+kf\,(\theta))}\\ \gamma_0 &= \frac{zkf\,(\theta)}{k+z\,(1+kf\,(\theta))}\end{aligned}$$

and $f(\theta)$ is such as:

$$Var\left(\varepsilon^P_{t-1,t-1}-\tilde{\varepsilon}_{t-1}\right) \equiv \frac{f\,(\theta)}{\beta\theta^2}$$

The parameter θ involved in the various expressions satisfies an equation given in the Appendix.

In addition to its own signals, the private sector observes the interest rate, which is set according to (12). The optimal policy parameter ρ satisfies the following relation:

$$\begin{aligned}\rho \;=\; & -\left[\begin{array}{c}(2+\kappa)\left[-\rho\,(k\,(1-\gamma_3)+(1-k)\,(1-\gamma_2))+\frac{(1-\rho)\theta}{\rho}\right]\\ +\tau k\,(1-x)\,\frac{\theta}{\rho}+(1+\kappa-\tau kx)\,(1-\gamma_2)\end{array}\right]\\ & \times (k\,(1-\gamma_0)+(1-k)\,(1-\gamma_4))\,\rho\\ & +\gamma_4+((2+\kappa)\,(1-\rho)+\tau k\,(1-x))\,\theta\,[1-((1-k)+kx)\,(\gamma_1\,(k\gamma_0+(1-k)\,\gamma_4)+(1-\gamma_1))]\end{aligned}$$

[3]The details of the computations can be found in the Appendix

To form its forecast $E_t^P \varepsilon_t$ the private sector obviously uses $\varepsilon_{t,t}^P$ with know variance β^{-1}. In addition, (12) shows that $\frac{1}{\rho}\left[\kappa r_t + \varepsilon_{t,t+1}^P - (1-\rho)\, s_t\right] = \varepsilon_{t,t}^{CB} - \frac{\left(\varepsilon_{t,t+1}^{CB} - \varepsilon_{t,t+1}^P\right)}{\rho}$ is a second signal available to the private sector. Since the private sector knows r_t and s_t, this expression is a second unbiased signal on ε_t, with variance $\left[\frac{1}{z} + \frac{1}{\rho^2}\frac{1}{k}\left(1+\frac{1}{z}\right)\right]\beta^{-1}$. Using Bayes rule, the private sector forecast is therefore:

$$E_t^P \varepsilon_t = \gamma_1 \varepsilon_{t,t}^P + (1-\gamma_1)\frac{1}{\rho}\left[\kappa r_t + \varepsilon_{t,t+1}^P - (1-\rho)\, s_t\right]$$

where:

$$\gamma_1 = \frac{1+z+\rho^2 k}{(1+z)(1+\rho^2 k)}$$

To form the forecast $E_t^P \varepsilon_{t+1}$, the private sector proceeds in the same way. A first signal is $\varepsilon_{t,t+1}^P$ with variance $(k\beta)^{-1}$ and the second signal is $-\kappa r_t + (1-\rho)s_t + \rho\varepsilon_{t,t}^P = \varepsilon_{t,t+1}^{CB} - \rho\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right)$ of variance : $\left[\frac{1}{kz} + \rho^2\left(1+\frac{1}{z}\right)\right]\beta^{-1}$ so that :

$$E_t^P \varepsilon_{t+1} = \gamma_2 \varepsilon_{t,t+1}^P + (1-\gamma_2)\left(-\kappa r_t + (1-\rho)s_t + \rho\varepsilon_{t,t}^P\right)$$

with:

$$\gamma_2 = \frac{1+\rho^2 k(1+z)}{(1+z)(1+\rho^2 k)}$$

An identical reasoning provides the private sector forecast of central bank signals:

$$\begin{aligned}
E_t^P \varepsilon_{t,t}^{CB} &= \gamma_5 \varepsilon_{t,t}^P + (1-\gamma_5)\frac{1}{\rho}\left[\kappa r_t - (1-\rho)\, s_t + \varepsilon_{t,t+1}^P\right] \\
E_t^P \varepsilon_{t,t+1}^{CB} &= \gamma_3 \varepsilon_{t,t+1}^P + (1-\gamma_3)\left[-\kappa r_t + (1-\rho)s_t + \rho\varepsilon_{t,t}^P\right]
\end{aligned}$$

with:

$$\begin{aligned}
\gamma_3 &= \frac{\rho^2 k}{1+\rho^2 k} \\
\gamma_5 &= \frac{1}{1+\rho^2 k}
\end{aligned}$$

Substituting (12) into (10) provides the inflation rate:

$$\begin{aligned}
\pi_t = {} & \left[\varepsilon_t - \left(\rho\varepsilon_{t,t}^{CB} + (1-\rho)\, s_t\right)\right] + \left[E_t^{CB}\varepsilon_{t+1} - E_t^P \varepsilon_{t+1}\right] \\
& + (2+\kappa)\left[E_t^P \varepsilon_{t+1} - \kappa E_t^P r_{t+1}\right] + \tau k\left[\left(\varepsilon_{t,t+1}^P - x E_t^P \varepsilon_{t+1} - (1-x)\, E_t^P s_{t+1}\right)\right]
\end{aligned}$$

where $E_t^P r_{t+1} = \frac{1}{\kappa} E_t^P\left[\rho\varepsilon_{t+1,t+1}^{CB} + (1-\rho)\, s_{t+1}\right]$. As under transparency, actual inflation is driven by forecast errors.

The first term is similar to the central bank forecast error $\left(\varepsilon_t - E_t^{CB}\varepsilon_t\right)$ in (15) but not identical since $E_t^{CB}\varepsilon_t \neq \rho\varepsilon_{t,t}^{CB} + (1-\rho)\, s_t$. Because central bank and private sector expectations are not aligned, the central bank decision on r_t must take into account the fact that the private sector will set the price level with a different interpretation of the signals. The second term is identical to what appears in (15). The same mechanism explains the fourth term, which can be further decomposed into $x(\varepsilon_{t,t+1}^P - E_t^P\varepsilon_{t+1})$ and $(1-x)\left(\varepsilon_{t,t+1}^P - E_t^P s_{t+1}\right)$. Both expressions reflect the fact that the private sector error forecasts carry over to the next period because the central bank takes them into account as it makes its policy decision.[4] The third term reflects the fact that the private sector no longer trusts the central bank for offseting the future shock.

5 Welfare Analysis

5.1 Welfare comparison

Since the model is time-invariant, welfare comparison only requires to consider the unconditional losses under transparency and under opacity at generic time t. The Appendix presents an analysis of the sign of the loss difference:

$$\beta\Delta L = \left(\beta E\pi_t^2\right)^{\text{op}} - \left(\beta E\pi_t^2\right)^{\text{tr}}$$

This analysis is only possible for some boundary values of the model's parameters $z \geq 0$, $k \in [0,1]$, and $\kappa \geq 0$. Numerical studies are summarized in Figure 1. The figure displays two curves that correspond to two values of κ[5]. The area below each curve corresponds to $\Delta L = L^{op} - L^{tr} < 0$, i.e. to the case where welfare is higher when the central bank does not reveal its interest rate path forecast.

The following proposition summarizes the results of this analysis:

Proposition 3 *When the central bank follows the optimal linear interest rule (7), ceteris paribus, transparency dominates when z is large and when k is large. The role of κ is ambiguous: when k is small, an increase in κ favors opacity while it favors transparency when k is large.*

The proposition raises the question of why opacity may ever dominate transparency and how this issue is affected by the model's parameters. We consider both issues.

Figure 1 about here

[4] The difference between $\varepsilon_{t,t+1}^P$ and $E_t^P\varepsilon_{t+1}$ is that the latter incorporates information gleaned from the interest rate decision of the central bank while $E_t^P s_{t+1}$ differs from $\varepsilon_{t,t+1}^P$ because it includes a noisy combination of past signals.

[5] The intercepts of these curves are provided in the Appendix

5.2 The usefulness of opacity

It might seem that more information is always desirable from a welfare viewpoint. The presence of herogeneous information suggests that the presumption does not necessarily apply. Indeed, the central bank and the private sector interact strategically under heterogeneous information: the central bank sets the short-term rate and the private sector sets the rest of the yield curve. Both actions contribute to determine the long-term interest rate, which is the channel through which monetary policy affects the economy. This interaction creates the possibility that opacity welfare dominates transparency. The conditions under which this happens are non-trivial and illustrate the subtlety of the issue of central bank transparency.

Two effects are involved, both of which reflect the fact that decisions are shaped by unavoidable forecast errors. On the one hand, under transparency, the central bank signals influence the private sector and, in turn, the central bank picks up signals from the private sector, this is the mirror effect. While expectations are aligned and inflation is always expected to be optimally at zero, actual inflation is never zero because unavoidable forecast errors, as Section 3 shows. Under opacity, on the other hand, not only do agents incorrectly forecast the shocks but they incorrectly forecast each other forecasts. This creates more reasons for inflation not to be zero, as can be seen in Section 4, but these forecast errors may well be systematically negatively correlated. When this is the case - it depends on parameter values as discussed below - the overall variability of inflation can be lower under opacity.

To see why, note that (11) can be rewritten as:

$$\pi_t = (2+\kappa)E_t^P\left(\varepsilon_{t+1} - \kappa r_{t+1}\right) - \kappa E_t^P \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n\left(1-\varsigma^2\right)} r_{t+n} - \psi_t \tag{16}$$

with:

$$\psi_t = \kappa r_t - \left(\varepsilon_t - E_t^P \varepsilon_{t+1}\right)$$

The term ψ_t can be interepreted as the "policy miss". It measures the private sector's perception of the extent to which the central bank fails to systematically achieve zero inflation. Put differently, $\pi_t + \psi_t$, which only depends on private sector expectations of future interest rates, is a measure of the extent to which the private sector trusts future central bank decisions.

Note that (16) is valid both regimes but the value of ψ_t depends on the regime. In particular, in the transparency regime, we have:

$$\beta Var^{tr}(\psi_t) = \frac{1}{z+k} + \frac{1}{kz\left(z+1\right)}$$

while in the opacity regime:

$$\begin{aligned}\beta Var^{op}(\psi_t) &= \frac{\gamma_2^2}{k}\left(1+\frac{1}{z}\right) + \frac{\left(\rho\gamma_2\right)^2}{z} + \frac{\left(\left(1-\rho\right)+k\left(1-\gamma_2\right)\rho\right)^2}{k} \\ &\quad + \left(1-k\right)\left(\left(1-\gamma_2\right)\rho\right)^2 + \left(1-\rho\right)^2 f\left(\theta\right)\end{aligned}$$

Under transparency, but not under opacity $E_t^P(\varepsilon_{t+1} - r_{t+1}) = E_t^P \pi_{t+1} = 0$, so opacity tends to raise inflation volatility. It follows that welfare is lower in the opacity regime unless $cov\left[(2+\kappa)E_t^P(\varepsilon_{t+1} - \kappa r_{t+1}) - \kappa E_t^P \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n(1-\varsigma^2)} r_{t+n}, \psi_t\right] > 0$. This is a necessary but not sufficient, condition for opacity to raise welfare. This leads to the following proposition:

Proposition 4 *(Creative opacity) A necessary condition for opacity to welfare-dominate transparency is that the private sector's own forecasts systematically offset the impact on inflation volatility of the central bank forecast errors.*

The proposition is based on the following observation. The term

$$E_t^P\left[(2+\kappa)(\varepsilon_{t+1} - \kappa r_{t+1}) - \kappa E_t^P \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n\left(1-\varsigma^2\right)} r_{t+n}\right]$$

represents the private sector forecast of future central bank's attempts to correct for past forecast errors while the policy miss does the same for current policy decisions.

This opens up the possibility that these corrections offset each other. For example, if it observes an interest rate r_t higher than what its own signals would justify, the private sector concludes that inflation will be too low. If it anticipates that the central bank will correct its mistake by lowering the interesting too much next period, it has the opposite effect on the current inflation, see (16). This combination stabilizes inflation.

5.3 The role of relative signal precision

A higher ratio z of central bank signal precision α to private sector signal precision β enhances the value of signals released by the cantral bank. Indeed, the publication of the interest rate path provides the private sector with a central bank signal that is more useful the more precise it is relative to its own signals. As z becomes smaller, the benefit from information disclosure declines because the private sector increasingly doubts any signal from the central bank, for good reason. Then, opacity may raise welfare when the private sector sets prices and the long-term interest rate so as to systematically offset the effects of potential large central bank forecast errors.

5.4 The role of early information precision

The parameter k, which ranges from 0 to 1, represents the precision of early signals relative to updated signals. By releasing in period t its forecast of the interest rate that it expects to set in period $t+1$, the central bank reveals its early signal $\varepsilon_{t,t+1}^{CB}$ of the shock ε_{t+1} expected in period $t+1$. Then, in period $t+1$, the central bank can decipher the early signal $\varepsilon_{t,t+1}^{P}$ received by the private sector in period t concerning the same shock ε_{t+1}. Since transparency makes this exchange of early signals possible, it is more desirable the more precise are these signals. Indeed, when $k = 0$, these signals become nearly useless. Why, then, even when $k = 0$, does there exist a high enough z to make transparency desirable as shown in Figure 1? [6]

Because the private sector always need to forecast ε_{t+1} to set inflation and the yield curve. Under opacity, the central bank relies on its early signal $\varepsilon^{P}_{t,,t+1}$ and on the interest rate r_t announced by the central bank, on the basis of $\varepsilon^{CB}_{t,t+1}$. When $k = 0$, these signals are infinitely imprecise but they are the only available information. When the central bank is generally better informed than the private sector – when z is large – it therefore helps the private sector to know $\varepsilon^{CB}_{t,t+1}$ and r_t. Under transparency, it does not assume that the central bank is misled and therefore still expects $E^{P}_{t}\pi_{t+1} = 0$.

5.5 The role of the demand elasticity of inflation

Figure 1 shows that the frontier beween transparency and opacity rotates clockwise when κ increases. This parameter captures the influence of demand on current inflation, see eq. (1). Since demand is driven by the private sector forecasts of future inflation and interest rate, as assumed in (2), κ is the crucial link between private forecasts and actual inflation. We have seen that the non-alignment of expectations under opacity lead to more forecast errors, which all impact inflation. *Prima facie*, therefore, inflation is more volatile under opacity. On the other hand, opacity raises welfare when some of these forecast errors are negatively correlated. When k is small, early signals are relatively imprecise and therefore lead the private sector to attach less weight to the central bank signals (for a given z). This makes negative correlation of forecasts errors more likely and explains why the frontier shifts up for low values of k.

6 Conclusion

In Gosselin et al. (2008), using a two period model, we provided a first characterization of the role of publication of interest rate forecasts by the central bank under the assumption that information is heterogeneous (the information sets of the central bank and of the previous sector are non-nested). This paper presents a generalization of the previous model to an infinite horizon. The generalization is only partial since we assume that signals about the shock are not available beyond the next period. This assumption is made for two reasons. First, technically, the extremely simple model that we use cannot be solved when the signals extend to infinity, or even beyond the next period. Second, the idea that we dispose of signals that extend far into the future seems to us more than far-fetched. Put differently, if we think of a

[6] The intercept of the curve on the vertical axis is $4 + 2\kappa$.

period as representing one year, we do have some information about the coming and the following year. Beyond that we move to the realm of Knightian uncertainty.

The present paper shows that most of the results from Gosselin et al. (2008) carry over to the more general setup, with some interesting differences. At the general level, transparency becomes relatively more desirable to in the extended model. The reason is that in the opacity regime, past forecast errors are carried over from one period to the next, which introduces infinite autoregressivity in the inflation process. By allowing expectations about the next period to be aligned, transparency eliminates this autoregressivity, which reduces the volatility of inflation. More precisely, with only two period, the central bank only recovers information about next-period private sector forecasts - the mirror effect. With an infinite horizon, this "next-period information" eventually becomes "current information" and transparency becomes more valuable.

Two final observations are required. First, it is relative, not absolute signal precision that matters for welfare ranking.[7] This matters because policy discussions of the merits or demerits of the alternative strategies often focus on absolute precision arguments. Quite intuitively, what matters is not how well is the central bank informed but how its information compares with the private sector inforomation. Second, opacity may dominate transparency for different reasons from those found by Morris and Shin (2002). Their mechanism involves a common knowledge effect, i.e. nested information sets, which is explicitely assumed here as we rely on heterogeneous information. Interestingly, in the transparency regime, the fact that expectations are aligned means that the private sector knows everything that the central bank knows. In that case, the signals become nested.

[7] Absolute precision matters for the size of the welfare losses, as the presence of β in our evaluations readily confirms.

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Appendix

6.1 Signal extraction: the form of $\tilde{\varepsilon}_{t-1}$

To extract the best possible signal $s_t = \varepsilon^P_{t-1,t} - \theta\left(\varepsilon^P_{t-1,t-1} - \tilde{\varepsilon}_{t-1}\right)$ about ε_t the central bank chooses $\tilde{\varepsilon}_{t-1}$ to minimize the noise from $\left(\varepsilon^P_{t-1,t-1} - \tilde{\varepsilon}_{t-1}\right)$. This optimal $\tilde{\varepsilon}_{t-1}$ must combine all central bank signals centered around $\varepsilon^P_{t-1,t-1}$. Of course, the central bank cannot observe $\varepsilon^P_{t-1,t-1}$ but it observes ε_{t-1} and s_{t-1}, both of which have the same mean as $\varepsilon^P_{t-1,t-1}$, so the best possible $\tilde{\varepsilon}_{t-1}$ is the following linear combination:

$$\tilde{\varepsilon}_{t-1} = w\varepsilon_{t-1} + (1-w)\, s_{t-1} = w\varepsilon_{t-1} + (1-w)\left(\varepsilon^P_{t-2,t-1} - \theta\left(\varepsilon^P_{t-2,t-2} - \tilde{\varepsilon}_{t-2}\right)\right)$$

with weights $0 < w < 1$. We do not use $\varepsilon^{CB}_{t-1,t-1}$ because this signal is always a worse indicator of $\varepsilon^P_{t-1,t-1}$ than ε_{t-1}.

Changing notations with $0 < x < 1$ such that $w = (1-k) + kx$, we have:

$$\tilde{\varepsilon}_{t-1} = ((1-k) + kx)\,\varepsilon_{t-1} + k(1-x)\left(\varepsilon^P_{t-2,t-1} - \theta\left(\varepsilon^P_{t-2,t-2} - \tilde{\varepsilon}_{t-2}\right)\right)$$

Importantly $\tilde{\varepsilon}_{t-1}$, which is chosen by the central bank, is also known to the private sector. To see why, note that we can use the recursivity in the previous equation to obtain:

$$\tilde{\varepsilon}_{t-1} = \sum_{n=0}^{\infty} \left[\theta k(1-x)\right]^n \left[((1-k) + kx)\,\varepsilon_{t-1-n} + k(1-x)\left(\varepsilon^P_{t-2-n,t-1-n} - \theta\varepsilon^P_{t-2-n,t-2-n}\right)\right]$$

which is a series of private sector signals.

In order to find the optimal weights x, we will use the following expression for $a_t = \varepsilon^P_{t-1,t-1} - \tilde{\varepsilon}_{t-1}$:

$$a_t = ((1-k) + kx)\left(\varepsilon^P_{t-1,t-1} - \varepsilon_{t-1}\right) + k(1-x)\left(\varepsilon^P_{t-1,t-1} - \varepsilon^P_{t-2,t-1}\right) + k(1-x)\,\theta a_{t-1}$$

The weight x that minimizes the unconditional variance of a_t is:

$$x = \frac{1}{2k^2\theta^2}\left(-1 - k\theta^2 + 2k^2\theta^2 + \sqrt{\left(1 + 2k\theta^2 - 4k^2\theta^2 + k^2\theta^4\right)}\right)$$

The choice of the positive root is imposed by the condition $w > 0$.

For further reference, we compute the resulting unconditional variance of a_t. Note that $\left(\varepsilon^P_{t-1,t-1} - \tilde{\varepsilon}_{t-1}\right)$ and $\left(\varepsilon^P_{t-2,t-2} - \tilde{\varepsilon}_{t-2}\right)$ have the same unconditional variances, so that:

$$\begin{aligned} Var\left(a_t\right) &= \frac{1 - k + k\left(\frac{1}{2k^2\theta^2}\left(-1 - k\theta^2 + 2k^2\theta^2 + \sqrt{\left(1 + k\theta^2 - 2k\theta\right)\left(k\theta^2 + 2k\theta + 1\right)}\right)\right)^2}{1 - (k\theta)^2\left(1 - \frac{1}{2k^2\theta^2}\left(-1 - k\theta^2 + 2k^2\theta^2 + \sqrt{\left(1 + k\theta^2 - 2k\theta\right)\left(k\theta^2 + 2k\theta + 1\right)}\right)\right)^2}\frac{1}{\beta} \\ &\equiv \frac{f(\theta)}{\theta^2}\frac{1}{\beta} \end{aligned}$$

6.2 Proof of (9)

To prove (9), we start with the autoregressive formula for s_t derived in the text:

$$s_t = \varepsilon^P_{t-1,t} - \theta\left(\varepsilon^P_{t-1,t-1} - [((1-k)+kx)\,\varepsilon_{t-1}]\right) + \theta k(1-x)\,s_{t-1}$$

This last relation can be iterated forward to find $E^P_t s_{t+n}$. Actually,

$$\begin{aligned}
E^P_t s_{t+n} &= (\theta k(1-x))^{n-1}\, s_{t+1} \\
&\quad + E^P_t \sum_{m=1}^{n-1} (\theta k(1-x))^{m-1}\left(\varepsilon^P_{t+n-m,t+n-m+1} - \theta\left(\varepsilon^P_{t+n-m,t+n-m} - ((1-k)+kx)\,\varepsilon_{t+n-m}\right)\right) \\
&= (\theta k(1-x))^{n-1}\, s_{t+1} - (\theta k(1-x))^{n-2}\,\theta E^P_t\left(\varepsilon^P_{t+1,t+1} - ((1-k)+kx)\,\varepsilon_{t+1}\right) \\
&= (\theta k(1-x))^{n-1}\, E^P_t s_{t+1} - (\theta k(1-x))^{n-2}\,\theta\left(k\varepsilon^P_{t,t+1} - kxE^P_t\varepsilon_{t+1}\right)
\end{aligned}$$

It follows that the infinite sum in (4) is:

$$\begin{aligned}
& -\kappa\xi \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n(1-\varsigma^2)} E^P_t s_{t+n} \\
= \ & -\kappa\xi \sum_{n=2}^{\infty} \left(\frac{1-\varsigma^{2n+2}}{\varsigma^n(1-\varsigma^2)}(\theta k(1-x))^{n-2}\right)\left[(\theta k(1-x))\,E^P_t s_{t+1} - \theta\left(k\varepsilon^P_{t,t+1} - kxE^P_t\varepsilon_{t+1}\right)\right]
\end{aligned}$$

and tedious computations show that:

$$-\kappa\xi \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n(1-\varsigma^2)} E^P_t s_{t+n} = \tau\left[\left(k\varepsilon^P_{t,t+1} - kxE^P_t\varepsilon_{t+1}\right) - k(1-x)\,E^P_t s_{t+1}\right] \qquad (17)$$

with:

$$\tau = \frac{\theta\kappa\xi}{(1-\varsigma^2)}\left(\frac{1}{\varsigma^2}\frac{1}{1-\frac{\theta k(1-x)}{\varsigma}} - \frac{\varsigma^2}{1-\varsigma\theta k(1-x)}\right)$$

as claimed.

6.3 Transparency: Proof that $\theta = 0$

We now deduce the precise form for the signal extracted by the central bank (i.e. the coefficient θ) in transparency. We will proceed in the following manner. We first compute the signal extracted at time $t+1$ by the central bank: $\pi_t - \varepsilon_t + \kappa r_t$. Recall the expression for the inflation dynamics derived in the text:

$$\begin{aligned}
\pi_t &= -(2+\kappa)\,\kappa E^P_t r_{t+1} - \kappa r_t + \varepsilon_t + (1+\kappa)\,E^P_t\varepsilon_{t+1} \\
&\quad + \tau\left(k\varepsilon^P_{t,t+1} - kxE^P_t\varepsilon_{t+1} - k(1-x)\,E^P_t s_{t+1}\right)
\end{aligned}$$

so that $\pi_t + \kappa r_t - \varepsilon_t$ is given by :

$$-\left(2+\kappa\right)\kappa E_t^P r_{t+1} + \left(1+\kappa\right) E_t^P \varepsilon_{t+1} + \tau\left(k\varepsilon_{t,t+1}^P - kxE_t^P\varepsilon_{t+1} - k\left(1-x\right)E_t^P s_{t+1}\right)$$

with $\tau = \frac{\theta\kappa\xi}{(1-\varsigma^2)}\left(\frac{1}{\varsigma^2}\frac{1}{1-\frac{\theta k(1-x)}{\varsigma}} - \frac{\varsigma^2}{1-\varsigma\theta k(1-x)}\right)$

The signal $\pi_t + \kappa r_t - \varepsilon_t$ can thus be rewritten as :

$$-\left(2+\kappa\right)\kappa E_t^P r_{t+1} + \left(1+\kappa-\tau kx\right) E_t^P \varepsilon_{t+1} + \tau k\varepsilon_{t,t+1}^P - \tau k\left(1-x\right)E_t^P s_{t+1}$$

By identification, we will get an equation for θ. To do so, we need to compute the various expectations involved in the previous expression. We start with $\kappa E_t^P r_{t+1}$. Since

$$\kappa r_{t+1} = \rho\varepsilon_{t+1,t+1}^{CB} + \left(1-\rho\right)\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - \tilde{\varepsilon}_t\right)\right) - \varepsilon_{t+1,t+2}^{CB}$$

we have then

$$\begin{aligned}
\kappa E_t^P r_{t+1} &= \rho E_t^P \varepsilon_{t+1,t+1}^{CB} + \left(1-\rho\right)\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right) \\
&= \rho\left[\left(\left(k + \left(1-k\right)\frac{z}{1+z}\right)\varepsilon_{t,t+1}^{CB} + \frac{(1-k)}{1+z}\varepsilon_{t,t+1}^P\right)\right] \\
&\quad + \left(1-\rho\right)\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right)
\end{aligned}$$

To compute $E_t^P\tilde{\varepsilon}_t$ we use the expression for $\tilde{\varepsilon}_t$ from the previous section of this Appendix:

$$\begin{aligned}
E_t^P\tilde{\varepsilon}_t &= \sum_{n=0}^{\infty}\left(\theta k\left(1-x\right)\right)^n\left(\left(\left(1-k\right)+kx\right)E_t^P\varepsilon_{t-n} + k\left(1-x\right)\left(\varepsilon_{t-1-n,t-n}^P - \theta\varepsilon_{t-1-n,t-1-n}^P\right)\right) \\
&= \left(\left(1-k\right)+kx\right)\left(E_t^P\varepsilon_t - \varepsilon_t\right) \\
&\quad + \sum_{n=0}^{\infty}\left(\theta k\left(1-x\right)\right)^n\left(\left(\left(1-k\right)+kx\right)\varepsilon_{t-n} + k\left(1-x\right)\left(\varepsilon_{t-1-n,t-n}^P - \theta\varepsilon_{t-1-n,t-1-n}^P\right)\right) \\
&= \left(\left(1-k\right)+kx\right)\left(E_t^P\varepsilon_t - \varepsilon_t\right) + \tilde{\varepsilon}_t
\end{aligned}$$

Using this expression, we get:

$$\begin{aligned}
\kappa E_t^P r_{t+1} &= \rho\left[\left(\left(k + \left(1-k\right)\frac{z}{1+z}\right)\varepsilon_{t,t+1}^{CB} + \frac{(1-k)}{1+z}\varepsilon_{t,t+1}^P\right)\right] \\
&\quad + \left(1-\rho\right)\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - \tilde{\varepsilon}_t\right)\right) + \left(1-\rho\right)\theta\left(\left(1-k\right)+kx\right)\left(E_t^P\varepsilon_t - \varepsilon_t\right) \\
&= \rho\left[\left(\left(k + \left(1-k\right)\frac{z}{1+z}\right)\varepsilon_{t,t+1}^{CB} + \frac{(1-k)}{1+z}\varepsilon_{t,t+1}^P\right)\right] \\
&\quad + \left(1-\rho\right)\theta\left(\left(1-k\right)+kx\right)\left[\frac{z}{1+z}\varepsilon_{t,t}^{CB} + \frac{1}{1+z}\varepsilon_{t,t}^P\right] \\
&\quad + \left(1-\rho\right)\left[s_{t+1} - \theta\left(\left(1-k\right)+kx\right)\varepsilon_t\right]
\end{aligned}$$

Similarly, we have:

$$\begin{aligned}
-\tau k(1-x)E_t^P s_{t+1} &= -\tau k(1-x)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-E_t^P\tilde{\varepsilon}_t\right)\right)\\
&= -\tau k(1-x)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)\right)-\tau k(1-x)\theta\left((1-k)+kx\right)\left(E_t^P\varepsilon_t-\varepsilon_t\right)\\
&= -\tau k(1-x)s_{t+1}-\tau k(1-x)\theta\left((1-k)+kx\right)\left(\frac{z}{1+z}\varepsilon_{t,t}^{CB}+\frac{1}{1+z}\varepsilon_{t,t}^P-\varepsilon_t\right)
\end{aligned}$$

On the other hand, $E_t^P\varepsilon_{t+1}=\frac{z}{1+z}\varepsilon_{t,t+1}^{CB}+\frac{1}{1+z}\varepsilon_{t,t+1}^P$. Moreover ε_t $\varepsilon_{t,t}^{CB}$, $\varepsilon_{t,t+1}^{CB}$as well as $\tilde{\varepsilon}_t$ are known to the central bank at time $t+1$. As a consequence, since $s_{t+1}=\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)$, the signal extracted by the central bank at this period is :

$$\left[\frac{1}{1+z}(1+\kappa-\tau kx)+\tau kx-(2+\kappa)\left(\rho\frac{(1-k)}{1+z}+(1-\rho)\right)\right]\varepsilon_{t,t+1}^P$$
$$-\theta\left((2+\kappa)(1-\rho)\left(\frac{((1-k)+kx)}{1+z}-1\right)-\tau k(1-x)\left(1-((1-k)+kx)\frac{1}{1+z}\right)\right)\varepsilon_{t,t}^P$$

extracted by the central bank is thus :

$$\varepsilon_{t,t+1}^P-\frac{\theta\left(-(2+\kappa)(1-\rho)-\tau k(1-x)\right)(k(1-x)+z)}{((2+\kappa)(\rho k+(\rho-1)z)-1+\tau kxz)}\varepsilon_{t,t}^P$$

By substracting $\tilde{\varepsilon}_t$ we get :

$$s_{t+1}=\varepsilon_{t,t+1}^P-\frac{\theta\left(-(2+\kappa)(1-\rho)-\tau k(1-x)\right)(k(1-x)+z)}{((2+\kappa)(\rho k+(\rho-1)z)-1+\tau kxz)}\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)$$

and by identification :

$$\theta=\frac{\theta\left(-(2+\kappa)(1-\rho)-\tau k(1-x)\right)(k(1-x)+z)}{((2+\kappa)(\rho k+(\rho-1)z)-1+\tau kxz)}$$

One obvious solution is $\theta=0$ leading to $s_{t+1}=\varepsilon_{t,t+1}^P$ as claimed before. We will see in the next section that other solutions (i.e. the possibility of multiple equilibria) are ruled out.

6.4 Transparency: the optimal interest rate

Given our expression for the inflation rate $\pi_t=-(2+\kappa)\kappa E_t^P r_{t+1}-\kappa r_t+\varepsilon_t+(1+\kappa)E_t^P\varepsilon_{t+1}+\tau\left(k\varepsilon_{t,t+1}^P-kxE_t^P\varepsilon_{t+1}-k(1-x)E_t^P s_{t+1}\right)$as well as our postulated expression for the interest rate $r_{t+1}=\kappa r_{t+1}=\rho\varepsilon_{t+1,t+1}^{CB}+(1-\rho)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)\right)-$

$\varepsilon_{t+1,t+2}^{CB}$ we can write the inflation dynamics at time t:

$$\begin{aligned}
\pi_t &= -(2+\kappa)\kappa E_t^P r_{t+1} - \kappa r_t + \varepsilon_t + (1+\kappa-\tau kx) E_t^P \varepsilon_{t+1} + \tau k \varepsilon_{t,t+1}^P - \tau k(1-x) E_t^P s_{t+1} \\
&= -(2+\kappa)\left[\begin{array}{c} \rho\left[\left(\left(k+(1-k)\frac{z}{1+z}\right)\varepsilon_{t,t+1}^{CB} + \frac{(1-k)}{1+z}\varepsilon_{t,t+1}^P\right)\right] \\ +(1-\rho)\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right) \end{array}\right] \\
&\quad + (1+\kappa-\tau kx)\left(\frac{z}{1+z}\varepsilon_{t,t+1}^{CB} + \frac{1}{1+z}\varepsilon_{t,t+1}^P\right) + \tau k \varepsilon_{t,t+1}^P \\
&\quad -\tau k(1-x)\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right) - \kappa r_t + \varepsilon_t \\
&= \left[\frac{1}{1+z}(1+\kappa-\tau kx) + \tau kx - (2+\kappa)\left(\rho\frac{(1-k)}{1+z} + (1-\rho)\right)\right]\varepsilon_{t,t+1}^P \\
&\quad + ((2+\kappa)(1-\rho) + \tau k(1-x))\theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right) \\
&\quad + \left[(1+\kappa-\tau kx)\frac{z}{1+z} - (2+\kappa)\rho\left(k + \frac{(1-k)z}{1+z}\right)\right]\varepsilon_{t,t+1}^{CB} - \kappa r_t + \varepsilon_t
\end{aligned}$$

Since r_t does not affect future inflation rates, it is thus optimal for the central bank to set it such as minimizing $E_t^{CB}\pi_t^2$, that is to set $E_t^{CB}\pi_t = 0$. Now, use that:

$$\begin{aligned}
E_t^{CB}\varepsilon_{t,t+1}^P &= E_t^{CB}\varepsilon_{t+1} = \varepsilon_{t,t+1}^{CB} \\
E_t^{CB}E_t^P\tilde{\varepsilon}_t &= E_t^{CB}\tilde{\varepsilon}_t
\end{aligned}$$

to directly obtain the following condition:

$$\begin{aligned}
r_t &= \frac{1}{\kappa}\left[\left(E_t^{CB}\varepsilon_t - E_t^{CB}\varepsilon_{t+1}\right) + ((2+\kappa)(1-\rho) + \tau k(1-x))\theta E_t^{CB}\left(\varepsilon_{t,t}^P - \tilde{\varepsilon}_t\right)\right] \\
&= \frac{1}{\kappa}\left[\left(E_t^{CB}\varepsilon_t - E_t^{CB}\varepsilon_{t+1}\right) + ((2+\kappa)(1-\rho) + \tau k(1-x))\right] \times \\
&\quad \times\theta E_t^{CB}\left(\varepsilon_{t,t}^P - [((1-k)+kx)\varepsilon_t + k(1-x)s_t]\right) \\
&= \frac{1}{\kappa}\left[\left(E_t^{CB}\varepsilon_t - E_t^{CB}\varepsilon_{t+1}\right) + ((2+\kappa)(1-\rho) + \tau k(1-x))\right] \times \\
&\quad \times\theta E_t^{CB}\left(k\varepsilon_{t-1,t}^P - [kx\varepsilon_t + k(1-x)s_t]\right) \\
&= \frac{1}{\kappa}\left[\left((1 - ((2+\kappa)(1-\rho) + \tau k(1-x))\theta kx\right)E_t^{CB}\varepsilon_t - E_t^{CB}\varepsilon_{t+1})\right] \\
&\quad +\frac{1}{\kappa}((2+\kappa)(1-\rho) + \tau k(1-x))\theta k\left(E_t^{CB}\varepsilon_{t-1,t}^P - (1-x)s_t\right) \\
&= \frac{1}{\kappa}\left[\left((1 - ((2+\kappa)(1-\rho) + \tau k(1-x))\theta kx\right)E_t^{CB}\varepsilon_t - E_t^{CB}\varepsilon_{t+1})\right] \\
&\quad +\frac{1}{\kappa}((2+\kappa)(1-\rho) + \tau k(1-x))\theta k\left(E_t^{CB}\varepsilon_{t-1,t}^P - (1-x)s_t\right)
\end{aligned}$$

We now compute the various expectations involved in the expression for r_t. Using Bayes rule, one has :

$$E_t^{CB}\varepsilon_t = \gamma_4\varepsilon_{t,t}^{CB} + (1-\gamma_4)\left(\varepsilon_{t-1,t}^P - \theta\left(\varepsilon_{t-1,t-1}^P - \tilde{\varepsilon}_{t-1}\right)\right)$$

$$E_t^{CB}\varepsilon_{t-1,t}^P = \gamma_0\varepsilon_{t,t}^{CB} + (1-\gamma_0)\left(\varepsilon_{t-1,t}^P - \theta\left(\varepsilon_{t-1,t-1}^P - \tilde{\varepsilon}_{t-1}\right)\right)$$

with

$$\begin{aligned}
\gamma_4 &= \frac{z\left(1+kf\left(\theta\right)\right)}{k+z\left(1+kf\left(\theta\right)\right)} \\
\gamma_0 &= \frac{zkf\left(\theta\right)}{k+z\left(1+kf\left(\theta\right)\right)}
\end{aligned}$$

so that ultimately:

$$\begin{aligned}
r_t &= \frac{1}{\kappa}\left(\left(1-\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)\theta kx\right)\gamma_4 + \left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)\theta k\gamma_0\right)\varepsilon_{t,t}^{CB} \\
&\quad + \frac{1}{\kappa}\left[\begin{array}{c} \left(1-\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)kx\right)\left(1-\gamma_4\right) \\ +\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)k\left(\left(1-\gamma_0\right)-\left(1-x\right)\right) \end{array}\right]s_t \\
&\quad - \frac{1}{\kappa}E_t^{CB}\varepsilon_{t+1}
\end{aligned}$$

As a consequence, one can identify the coefficient ρ as satisfying :

$$\rho = \left(\left(1-\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)kx\right)\gamma_4 + \left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)k\gamma_0\right)$$

This equation, supplemented with the previous condition for θ,

$$\theta = \frac{\theta\left(-\left(2+\kappa\right)\left(1-\rho\right)-\tau k\left(1-x\right)\right)\left(k\left(1-x\right)+z\right)}{\left(\left(2+\kappa\right)\left(\rho k+\left(\rho-1\right)z\right)-1+\tau kxz\right)}$$

as well as the expression for the coefficients γ_4 and γ_0

$$\begin{aligned}
\gamma_4 &= \frac{z\left(1+kf\left(\theta\right)\right)}{k+z\left(1+kf\left(\theta\right)\right)} \\
\gamma_0 &= \frac{zkf\left(\theta\right)}{k+z\left(1+kf\left(\theta\right)\right)}
\end{aligned}$$

yields a system of equations that can be solved as functions of the parameters k, z and κ. It can be seen numerically that at least for reasonnable values of these parameters, the only real solution for θ is given by $\theta = 0$, that is $s_{t+1} = \varepsilon_{t,t+1}^P$ the full mirror effect, as claimed in the previous paragraph. Note that an immediate consequence of $\theta = 0$ is that $r_t = \frac{1}{\kappa}\left(E_t^{CB}\varepsilon_t - E_t^{CB}\varepsilon_{t+1}\right)$

6.5 Opacity: Determination of the coefficients θ

Using the forms for the expectation derived in the text, we can now deduce the precise form for the signal extracted by the central bank (i.e. the coefficient θ), as well as the coefficient ρ. We will proceed in the following manner. We first compute

$-\left(2+\kappa\right)\kappa E_t^P r_{t+1}+\left(1+\kappa-\tau k x\right)E_t^P\varepsilon_{t+1}+\tau\left(k\varepsilon_{t,t+1}^P-k\left(1-x\right)E_t^P s_{t+1}\right)$ which is the signal extracted by the central bank. By identification, this will lead to an equation for θ. The expression for θ will then allow to find the cental bank expectations, and then ultimately the interest rate that has to cancel the central bank expectation of inflation. This ultimately provides an equation for ρ. We start by computing $\kappa E_t^P r_{t+1}$. Since

$$\kappa r_{t+1}=\rho\varepsilon_{t+1,t+1}^{CB}+\left(1-\rho\right)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)\right)-\varepsilon_{t+1,t+2}^{CB}$$

we have :

$$\begin{aligned}
\kappa E_t^P r_{t+1} &= \rho E_t^P\varepsilon_{t+1,t+1}^{CB}+\left(1-\rho\right)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-E_t^P\tilde{\varepsilon}_t\right)\right)\\
&= \rho\left[\left(k\gamma_3+\left(1-k\right)\gamma_2\right)\varepsilon_{t,t+1}^P+\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)\left(\varepsilon_{t,t+1}^{CB}-\rho\left(\varepsilon_{t,t}^{CB}-\varepsilon_{t,t}^P\right)\right)\right]\\
&\quad+\left(1-\rho\right)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-E_t^P\tilde{\varepsilon}_t\right)\right)
\end{aligned}$$

To compute $E_t^P\tilde{\varepsilon}_t$ recall that we had :

$$\tilde{\varepsilon}_t=\sum_{n=0}^{\infty}\left(\theta k\left(1-x\right)\right)^n\left(\left(\left(1-k\right)+kx\right)\varepsilon_{t-n}+k\left(1-x\right)\left(\varepsilon_{t-1-n,t-n}^P-\theta\varepsilon_{t-1-n,t-1-n}^P\right)\right)$$

so that :

$$\begin{aligned}
E_t^P\tilde{\varepsilon}_t &= \sum_{n=0}^{\infty}\left(\theta k\left(1-x\right)\right)^n\left(\left(\left(1-k\right)+kx\right)E_t^P\varepsilon_{t-n}+k\left(1-x\right)\left(\varepsilon_{t-1-n,t-n}^P-\theta\varepsilon_{t-1-n,t-1-n}^P\right)\right)\\
&= \left(\left(1-k\right)+kx\right)\left(E_t^P\varepsilon_t-\varepsilon_t\right)\\
&\quad+\sum_{n=0}^{\infty}\left(\theta k\left(1-x\right)\right)^n\left(\left(\left(1-k\right)+kx\right)\varepsilon_{t-n}+k\left(1-x\right)\left(\varepsilon_{t-1-n,t-n}^P-\theta\varepsilon_{t-1-n,t-1-n}^P\right)\right)\\
&= \left(\left(1-k\right)+kx\right)\left(E_t^P\varepsilon_t-\varepsilon_t\right)+\tilde{\varepsilon}_t
\end{aligned}$$

and :

$$\begin{aligned}
\kappa E_t^P r_{t+1} &= \rho E_t^P\varepsilon_{t+1,t+1}^{CB}+\left(1-\rho\right)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-E_t^P\tilde{\varepsilon}_t\right)\right)\\
&= \rho\left[\left(k\gamma_3+\left(1-k\right)\gamma_2\right)\varepsilon_{t,t+1}^P+\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)\left(\varepsilon_{t,t+1}^{CB}-\rho\left(\varepsilon_{t,t}^{CB}-\varepsilon_{t,t}^P\right)\right)\right]\\
&\quad+\left(1-\rho\right)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)\right)+\left(1-\rho\right)\theta\left(\left(1-k\right)+kx\right)\left(E_t^P\varepsilon_t-\varepsilon_t\right)\\
&= \rho\left[\left(k\gamma_3+\left(1-k\right)\gamma_2\right)\varepsilon_{t,t+1}^P+\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)\left(\varepsilon_{t,t+1}^{CB}-\rho\left(\varepsilon_{t,t}^{CB}-\varepsilon_{t,t}^P\right)\right)\right]\\
&\quad+\left(1-\rho\right)\theta\left(\left(1-k\right)+kx\right)\left[\gamma_1\varepsilon_{t,t}^P+\left(1-\gamma_1\right)\left(\varepsilon_{t,t}^{CB}+\frac{\left(\varepsilon_{t,t+1}^P-\varepsilon_{t,t+1}^{CB}\right)}{\rho}\right)\right]\\
&\quad+\left(1-\rho\right)\left[s_{t+1}-\theta\left(\left(1-k\right)+kx\right)\varepsilon_t\right]
\end{aligned}$$

Similarly to the transparency case, we can also compute $-\tau k\left(1-x\right)E_t^P s_{t+1}$:

$$\begin{aligned}
-\tau k\left(1-x\right)E_t^P s_{t+1} &= -\tau k\left(1-x\right)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-E_t^P\tilde{\varepsilon}_t\right)\right)\\
&= -\tau k\left(1-x\right)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)\right)-\tau k\left(1-x\right)\theta\left(\left(1-k\right)+kx\right)\left(E_t^P\varepsilon_t-\varepsilon_t\right)\\
&= -\tau k\left(1-x\right)s_{t+1}\\
&\quad -\tau k\left(1-x\right)\theta\left(\left(1-k\right)+kx\right)\\
&\quad \times\left(\gamma_1\varepsilon_{t,t}^P+\left(1-\gamma_1\right)\left(\varepsilon_{t,t}^{CB}+\frac{\left(\varepsilon_{t,t+1}^P-\varepsilon_{t,t+1}^{CB}\right)}{\rho}\right)-\varepsilon_t\right)
\end{aligned}$$

On the other hand, $E_t^P\varepsilon_{t+1}=\gamma_2\varepsilon_{t,t+1}^P+\left(1-\gamma_2\right)\left(\varepsilon_{t,t+1}^{CB}-\left(\rho\right)\left(\varepsilon_{t,t}^{CB}-\varepsilon_{t,t}^P\right)\right)$ and as well as ε_t and $\tilde{\varepsilon}_t$ are known to the central bank at time $t+1$. As a consequence, since $s_{t+1}=\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)$, the signal extracted by the central bank at this period is :

$$\begin{aligned}
&\left[\begin{array}{c}\left(1+\kappa-\tau kx\right)\gamma_2-\left(2+\kappa\right)\left(\rho\left(k\gamma_3+\left(1-k\right)\gamma_2\right)+\left(1-\rho\right)\right)\\ +\tau k\left(x-\left(1-x\right)\theta\left(\left(1-k\right)+kx\right)\frac{\left(1-\gamma_1\right)}{\rho}\right)\end{array}\right]\varepsilon_{t,t+1}^P\\
&+\left[\begin{array}{c}\left(1+\kappa-\tau kx\right)\left(1-\gamma_2\right)-\left(2+\kappa\right)\left(\rho\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)-\theta\frac{\left(1-\rho\right)}{\rho}\right)\\ +\tau k\left(1-x\right)\theta\frac{1-\left(\left(1-k\right)+kx\right)\gamma_1}{\rho}\end{array}\right]\rho\varepsilon_{t,t}^P\\
&-\left(2+\kappa\right)\left(1-\rho\right)\theta\left(\left(1-k\right)+kx\right)\left[\gamma_1\varepsilon_{t,t}^P+\left(1-\gamma_1\right)\left(\frac{\varepsilon_{t,t+1}^P}{\rho}\right)\right]\\
=&\left[\begin{array}{c}\left(1+\kappa-\tau kx\right)\gamma_2+\tau kx-\tau k\left(1-x\right)\theta\left(\left(1-k\right)+kx\right)\frac{\left(1-\gamma_1\right)}{\rho}\\ -\left(2+\kappa\right)\left(\left(\rho\left(k\gamma_3+\left(1-k\right)\gamma_2\right)+\left(1-\rho\right)\right)+\left(1-\rho\right)\theta\left(\left(1-k\right)+kx\right)\frac{\left(1-\gamma_1\right)}{\rho}\right)\end{array}\right]\varepsilon_{t,t+1}^P\\
&+\left[\begin{array}{c}\left(1+\kappa-\tau kx\right)\left(1-\gamma_2\right)+\tau k\left(1-x\right)\theta\frac{1-\left(\left(1-k\right)+kx\right)\gamma_1}{\rho}\\ -\left(2+\kappa\right)\left(\begin{array}{c}\left(\rho\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)-\theta\frac{\left(1-\rho\right)}{\rho}\right)\\ +\left(1-\rho\right)\theta\left(\left(1-k\right)+kx\right)\frac{\gamma_1}{\rho}\end{array}\right)\end{array}\right]\rho\varepsilon_{t,t}^P
\end{aligned}$$

By substracting $\tilde{\varepsilon}_t$ and normalizing, we recover the signal s_{t+1} :

$$s_{t+1}=\varepsilon_{t,t+1}^P+\frac{N}{D}\rho\left(\varepsilon_{t,t}^P-\tilde{\varepsilon}_t\right)$$

so that :

$$\theta=-\frac{N}{D}\rho$$

with:

$$
\begin{aligned}
N &= (1+\kappa-\tau kx)(1-\gamma_2)+\tau k(1-x)\theta\frac{1-((1-k)+kx)\gamma_1}{\rho} \\
&\quad -(2+\kappa)\left(\left(\rho\left(k(1-\gamma_3)+(1-k)(1-\gamma_2)\right)-\theta\frac{(1-\rho)}{\rho}\right)+(1-\rho)\theta((1-k)+kx)\frac{\gamma_1}{\rho}\right) \\
D &= (1+\kappa-\tau kx)\gamma_2+\tau kx-\tau k(1-x)\theta((1-k)+kx)\frac{(1-\gamma_1)}{\rho} \\
&\quad -(2+\kappa)\left((\rho(k\gamma_3+(1-k)\gamma_2)+(1-\rho))+(1-\rho)\theta((1-k)+kx)\frac{(1-\gamma_1)}{\rho}\right)
\end{aligned}
$$

6.6 Opacity: Determination of the coefficients ρ

We now turn to the determination of the optimal interest rate at time t. To do so, as in the transparency case, we start with the inflation rate :

$$
\begin{aligned}
\pi_t &= -(2+\kappa)\kappa E_t^P r_{t+1}-\kappa r_t+\varepsilon_t+(1+\kappa-\tau kx)E_t^P\varepsilon_{t+1}+\tau k\varepsilon_{t,t+1}^P-\tau k(1-x)E_t^P s_{t+1} \\
&= -(2+\kappa)\left[\begin{array}{c}\rho\left[(k\gamma_3+(1-k)\gamma_2)\varepsilon_{t,t+1}^P+\left(\begin{array}{c}k(1-\gamma_3)\\+(1-k)(1-\gamma_2)\end{array}\right)\left(\varepsilon_{t,t+1}^{CB}-\rho\left(\varepsilon_{t,t}^{CB}-\varepsilon_{t,t}^P\right)\right)\right]\\+(1-\rho)\left(\varepsilon_{t,t+1}^P-\theta\left(\varepsilon_{t,t}^P-E_t^P\tilde{\varepsilon}_t\right)\right)\end{array}\right] \\
&\quad +(1+\kappa-\tau kx)\left(\gamma_2\varepsilon_{t,t+1}^P+(1-\gamma_2)\left(\varepsilon_{t,t+1}^{CB}-(\rho)\left(\varepsilon_{t,t}^{CB}-\varepsilon_{t,t}^P\right)\right)\right) \\
&\quad +\tau k\varepsilon_{t,t+1}^P-\tau k(1-x)E_t^P s_{t+1}-\kappa r_t+\varepsilon_t
\end{aligned}
$$

The optimal κr_t is found to cancel the central bank expectation of π_t. The reason for this is similar to the transparency case and is the following. The subsequent inflation rates:

$$
\pi_{t+i}=-(2+\kappa)\kappa E_{t+i}^P r_{t+i+1}+(1+\kappa)E_{t+i}^P\varepsilon_{t+i+1}-\kappa r_{t+i}+\varepsilon_{t+i}+\tau\left(k\varepsilon_{t+i,t+i+1}^P-k(1-x)E_{t+i}^P s_{t+i+1}\right)
$$

do not involve r_t. Actually, the expectations E_{t+i}^P, will involve the private signals on future shocks, as well as the future policy instruments r_{t+i}. These last ones will be set later independently from r_t. As a consequence, π_{t+i} will not depend on r_t, so that the central bank optimizing the intertemporal loss at time t can just aim at minimizing $E_t^{CB}\pi_t^2$. Morover, considering that in the equilibrium, the private sector sets its expectations given the linear form for the interest rate, π_t depends linearly on r_t : directly via $-\kappa r_t$ and through the private sector expectations using r_t as a signal. As a consequence, the optimal choice for the central bank is to set r_t so that

$E_t^{CB}\pi_t = 0$. We are thus led to:

$$\begin{aligned}
\kappa r_t &= -(2+\kappa) E_t^{CB}\left[\begin{array}{c} \rho\left[(k\gamma_3 + (1-k)\gamma_2)\varepsilon_{t,t+1}^P + \left(\begin{array}{c} k(1-\gamma_3) \\ +(1-k)(1-\gamma_2)\end{array}\right)\left(\varepsilon_{t,t+1}^{CB} - \rho\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right)\right)\right] \\ +(1-\rho)\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right)\end{array}\right. \\
&\quad + (1+\kappa-\tau kx) E_t^{CB}\left(\gamma_2\varepsilon_{t,t+1}^P + (1-\gamma_2)\left(\varepsilon_{t,t+1}^{CB} - (\rho)\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right)\right)\right) + E_t^{CB}\varepsilon_t \\
&\quad + E_t^{CB}\left(\tau k\varepsilon_{t,t+1}^P - \tau k(1-x)E_t^P s_{t+1}\right) \\
&= -\varepsilon_{t,t+1}^{CB} - (2+\kappa)E_t^{CB}\left[\begin{array}{c} -\rho\left[(k(1-\gamma_3)+(1-k)(1-\gamma_2))\left(\rho\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right)\right)\right] \\ -(1-\rho)\left(\theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right)\end{array}\right] \\
&\quad -(1+\kappa-\tau kx)E_t^{CB}(1-\gamma_2)\rho\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right) + E_t^{CB}\varepsilon_t \\
&\quad -\tau k(1-x)E_t^{CB}\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right) \\
&= -\varepsilon_{t,t+1}^{CB} - E_t^{CB}\left[\begin{array}{c} (2+\kappa)\left[-\rho(k(1-\gamma_3)+(1-k)(1-\gamma_2)) + \frac{(1-\rho)\theta}{\rho}\right] \\ +(1+\kappa-\tau kx)(1-\gamma_2)\end{array}\right]\rho\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right) \\
&\quad + (2+\kappa)(1-\rho)\left(\theta\left(\varepsilon_{t,t}^{CB} - E_t^{CB}E_t^P\tilde{\varepsilon}_t\right)\right) + \left(-\tau k(1-x)E_t^{CB}\left(\varepsilon_{t,t+1}^P - \theta\left(\varepsilon_{t,t}^P - E_t^P\tilde{\varepsilon}_t\right)\right)\right) \\
&\quad + E_t^{CB}\varepsilon_t \\
&= -\varepsilon_{t,t+1}^{CB} - E_t^{CB}\left[\begin{array}{c} (2+\kappa)\left[-\rho(k(1-\gamma_3)+(1-k)(1-\gamma_2)) + \frac{(1-\rho)\theta}{\rho}\right] \\ +\tau k(1-x)\frac{\theta}{\rho} + (1+\kappa-\tau kx)(1-\gamma_2)\end{array}\right]\rho\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right) \\
&\quad + ((2+\kappa)(1-\rho)+\tau k(1-x))\left(\theta\left(\varepsilon_{t,t}^{CB} - E_t^{CB}E_t^P\left[((1-k)+kx)\varepsilon_t + k(1-x)s_t\right]\right)\right) + E_t^{CB}\varepsilon_t
\end{aligned}$$

At this point we need to compute $E_t^{CB}E_t^P\left[((1-k)+kx)\varepsilon_t + k(1-x)s_t\right]$. Recall that at time t, $\tilde{\varepsilon}_{t-1}$ and thus s_t are known to the private sector. Thus,

$$\begin{aligned}
& E_t^{CB}E_t^P\left[((1-k)+kx)\varepsilon_t + k(1-x)s_t\right] \\
=\ & \left[((1-k)+kx)E_t^{CB}E_t^P\varepsilon_t + k(1-x)s_t\right] \\
=\ & E_t^{CB}\left[((1-k)+kx)\left(\gamma_1\varepsilon_{t,t}^P + (1-\gamma_1)\left(\varepsilon_{t,t}^{CB} + \frac{\left(\varepsilon_{t,t+1}^P - \varepsilon_{t,t+1}^{CB}\right)}{(\rho)}\right)\right) + k(1-x)s_t\right] \\
=\ & ((1-k)+kx)(\gamma_1(k\gamma_0 + (1-k)\gamma_4) + (1-\gamma_1))\varepsilon_{t,t}^{CB} \\
& + \left[((1-k)+kx)\gamma_1(k(1-\gamma_0) + (1-k)(1-\gamma_4)) + k(1-x)\right]s_t
\end{aligned}$$

So that we can ultimately rewrite :

$$\kappa r_t = -\varepsilon_{t,t+1}^{CB} + \rho\varepsilon_{t,t}^{CB} + (1-\rho)s_t$$

where ρ satisfies :

$$\begin{aligned}
\rho &= -\left[\left[\begin{array}{c} (2+\kappa)\left[-\rho(k(1-\gamma_3)+(1-k)(1-\gamma_2)) + \frac{(1-\rho)\theta}{\rho}\right] \\ +\tau k(1-x)\frac{\theta}{\rho} + (1+\kappa-\tau kx)(1-\gamma_2)\end{array}\right]\rho(k(1-\gamma_0)+(1-k)(1-\gamma_4))\right] \\
&\quad +\gamma_4 + ((2+\kappa)(1-\rho)+\tau k(1-x))\theta\left[1 - ((1-k)+kx)(\gamma_1(k\gamma_0 + (1-k)\gamma_4) + (1-\gamma_1))\right]
\end{aligned}$$

6.7 Inflation dynamics and the Loss

Using the form of the optimal interest rate, we can ultimately turn to the inflation dynamics. To do so, we first give three usefull expectations :

$$\varepsilon_t - E_t^{CB}\varepsilon_t = \gamma_4\left(\varepsilon_t - \varepsilon_{t,t}^{CB}\right) + (1-\gamma_4)\left(\varepsilon_t - \varepsilon_{t-1,t}^{P} + \theta\left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1}\right)\right)$$

$$\begin{aligned}\left(\varepsilon_{t,t}^{P} - E_t^{CB}\varepsilon_{t,t}^{P}\right) &= \varepsilon_{t,t}^{P} - \left[(k\gamma_0 + (1-k)\gamma_4)\,\varepsilon_{t,t}^{CB} + (k(1-\gamma_0) + (1-k)(1-\gamma_4))\,s_t\right] \\ &= \left(\varepsilon_{t,t}^{P} - \varepsilon_{t-1,t}^{P}\right) \\ &\quad + \left((k\gamma_0 + (1-k)\gamma_4)\left(\varepsilon_{t-1,t}^{P} - \varepsilon_{t,t}^{CB}\right) + \theta\begin{pmatrix} k(1-\gamma_0) \\ +(1-k)(1-\gamma_4)\end{pmatrix}\left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1}\right)\right)\end{aligned}$$

$$\begin{aligned}\left(E_t^{CB}E_t^{P}\varepsilon_t - E_t^{P}\varepsilon_t\right) &= (\gamma_1(k\gamma_0 + (1-k)\gamma_4) + (1-\gamma_1))\,\varepsilon_{t,t}^{CB} \\ &\quad +\gamma_1(k(1-\gamma_0) + (1-k)(1-\gamma_4))\,s_t \\ &\quad -\left(\gamma_1\varepsilon_{t,t}^{P} + (1-\gamma_1)\left(\varepsilon_{t,t}^{CB} + \frac{\left(\varepsilon_{t,t+1}^{P} - \varepsilon_{t,t+1}^{CB}\right)}{\rho}\right)\right) \\ &= \gamma_1\left((k\gamma_0 + (1-k)\gamma_4)\,\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^{P} + \begin{pmatrix} k(1-\gamma_0) \\ +(1-k)(1-\gamma_4)\end{pmatrix} s_t\right) \\ &\quad -(1-\gamma_1)\frac{\left(\varepsilon_{t,t+1}^{P} - \varepsilon_{t,t+1}^{CB}\right)}{\rho} \\ &= \gamma_1\begin{pmatrix} (k\gamma_0 + (1-k)\gamma_4)\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t-1,t}^{P}\right) \\ -\theta(k(1-\gamma_0) + (1-k)(1-\gamma_4))\left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1}\right)\end{pmatrix} \\ &\quad -(1-\gamma_1)\frac{\left(\varepsilon_{t,t+1}^{P} - \varepsilon_{t,t+1}^{CB}\right)}{\rho} - \gamma_1\left(\varepsilon_{t,t}^{P} - \varepsilon_{t-1,t}^{P}\right)\end{aligned}$$

As a consequence, some computations lead to the following formula:

$$\begin{aligned}\pi_t &= A\left(\varepsilon_{t,t+1}^{CB} - \varepsilon_{t,t+1}^{P}\right) + B\left(\varepsilon_{t-1,t}^{P} - \varepsilon_{t,t}^{CB}\right) + C\left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1}\right) + D\left(\varepsilon_{t,t}^{P} - \varepsilon_{t-1,t}^{P}\right) \\ &\quad +\gamma_4\left(\varepsilon_t - \varepsilon_{t,t}^{CB}\right) + (1-\gamma_4)\left(\varepsilon_t - \varepsilon_{t-1,t}^{P}\right) \\ &= A\left(\varepsilon_{t,t+1}^{CB} - \varepsilon_{t,t+1}^{P}\right) + (B+\gamma_4)\left(\varepsilon_t - \varepsilon_{t,t}^{CB}\right) + (B - (1-k)D - (1-\gamma_4))\left(\varepsilon_{t-1,t}^{P} - \varepsilon_t\right) \\ &\quad +C\left(\varepsilon_{t-1,t-1}^{P} - \tilde{\varepsilon}_{t-1}\right) + D(1-k)\left(\hat{\varepsilon}_{t,t}^{P} - \varepsilon_t\right)\end{aligned}$$

where the various coefficients are given by:

$$A = \begin{pmatrix} 1 - (2+\kappa)\rho(k(1-\gamma_3) + (1-k)(1-\gamma_2)) + (1+\kappa-\tau kx)(1-\gamma_2) \\ +\frac{(1-\gamma_1)}{\rho}((2+\kappa)(1-\rho) + \tau k(1-x))\theta((1-k)+kx)\end{pmatrix}$$

$$B = \begin{bmatrix} (2+\kappa)\left[-\rho\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)+\frac{(1-\rho)\theta}{\rho}\right] \\ +\tau k\left(1-x\right)\frac{\theta}{\rho}+\left(1+\kappa-\tau kx\right)\left(1-\gamma_2\right) \end{bmatrix}\rho\left(k\gamma_0+\left(1-k\right)\gamma_4\right)$$
$$-\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)\theta\left(\left(1-k\right)+kx\right)\gamma_1\left(k\gamma_0+\left(1-k\right)\gamma_4\right)$$

$$C = \left(\begin{array}{c} \left(1-\gamma_4\right)\theta-\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)\theta\left(\left(1-k\right)+kx\right)\gamma_1\left(\theta\left(k\left(1-\gamma_0\right)+\left(1-k\right)\left(1-\gamma_4\right)\right)\right) \\ +\begin{bmatrix} (2+\kappa)\left[-\rho\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)+\frac{(1-\rho)\theta}{\rho}\right] \\ +\tau k\left(1-x\right)\frac{\theta}{\rho}+\left(1+\kappa-\tau kx\right)\left(1-\gamma_2\right) \end{bmatrix}\rho\theta\left(k\left(1-\gamma_0\right)+\left(1-k\right)\left(1-\gamma_4\right)\right) \end{array} \right.$$

$$D = \begin{bmatrix} (2+\kappa)\left[-\rho\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)+\frac{(1-\rho)\theta}{\rho}\right] \\ +\tau k\left(1-x\right)\frac{\theta}{\rho}+\left(1+\kappa-\tau kx\right)\left(1-\gamma_2\right) \end{bmatrix}\rho$$
$$-\gamma_1\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)\theta\left(\left(1-k\right)+kx\right)$$

The previous expression for the inflation allows to compute easily the loss at time t. Actually, it is an expansion in terms of independent random variables. As a consequence, the Loss is given by :

$$\begin{aligned} \left(\beta E\pi_t^2\right)^{\text{opacity}} &= \frac{A^2}{k}\left(1+\frac{1}{z}\right)+\frac{\left(B+\gamma_4\right)^2}{z}+\frac{\left(B-\left(1-k\right)D-\left(1-\gamma_4\right)\right)^2}{k} \\ &\quad +C^2\frac{f\left(\theta\right)}{\theta^2}+D^2\left(1-k\right)-\left(\frac{1}{kz\left(1+z\right)}+\frac{1}{z+k}\right) \end{aligned}$$

6.8 Welfare comparison

The study the welfare difference one has to study the sign of

$$\begin{aligned} \beta\Delta L_t &= \left(\beta E\pi_t^2\right)^{\text{opacity}}-\left(\beta E\pi_t^2\right)^{\text{tranparency}} \\ &= \frac{A^2}{k}\left(1+\frac{1}{z}\right)+\frac{\left(B+\gamma_4\right)^2}{z}+\frac{\left(B-\left(1-k\right)D-\left(1-\gamma_4\right)\right)^2}{k} \\ &\quad +C^2\frac{f\left(\theta\right)}{\theta^2}+D^2\left(1-k\right)- \end{aligned}$$

where the parameters satisfiy the following set of equations :

$$\rho = -\begin{bmatrix} (2+\kappa)\left[-\rho\left(k\left(1-\gamma_3\right)+\left(1-k\right)\left(1-\gamma_2\right)\right)+\frac{(1-\rho)\theta}{\rho}\right]+\tau k\left(1-x\right)\frac{\theta}{\rho} \\ +\left(1+\kappa-\tau kx\right)\left(1-\gamma_2\right) \end{bmatrix}$$
$$\times\left(k\left(1-\gamma_0\right)+\left(1-k\right)\left(1-\gamma_4\right)\right)\rho$$
$$+\gamma_4+\left(\left(2+\kappa\right)\left(1-\rho\right)+\tau k\left(1-x\right)\right)\theta\left[1-\left(\left(1-k\right)+kx\right)\left(\gamma_1\left(k\gamma_0+\left(1-k\right)\gamma_4\right)+\left(1-\gamma_1\right)\right)\right]$$

$$\theta = -\frac{\left[\begin{array}{c} (1+\kappa-\tau kx)(1-\gamma_2)-(2+\kappa)\begin{pmatrix} \rho\left(k(1-\gamma_3)+(1-k)(1-\gamma_2)\right) \\ -\theta^{\frac{(1-\rho)}{\rho}}+(1-\rho)\,\theta\left((1-k)+kx\right)\frac{\gamma_1}{\rho} \end{pmatrix} \\ +\tau k(1-x)\,\theta^{\frac{1-((1-k)+kx)\gamma_1}{\rho}} \end{array}\right]}{\left[\begin{array}{c} (1+\kappa-\tau kx)\gamma_2-(2+\kappa)\begin{pmatrix} \left(\rho\left(k\gamma_3+(1-k)\gamma_2\right)+(1-\rho)\right) \\ +(1-\rho)\,\theta\left((1-k)+kx\right)\frac{(1-\gamma_1)}{\rho} \end{pmatrix} \\ +\tau kx-\tau k(1-x)\,\theta\left((1-k)+kx\right)\frac{(1-\gamma_1)}{\rho} \end{array}\right]}\rho$$

$$\begin{aligned}
\gamma_0 &= \frac{zkf(\theta)}{k+z\left(1+kf(\theta)\right)} \\
\gamma_1 &= \frac{\frac{1}{z}+\frac{1}{(\rho)^2}\frac{1}{k}\left(1+\frac{1}{z}\right)}{1+\frac{1}{z}+\frac{1}{(\rho)^2}\frac{1}{k}\left(1+\frac{1}{z}\right)} \\
\gamma_2 &= \frac{\frac{1}{kz}+(\rho)^2\left(1+\frac{1}{z}\right)}{\frac{1}{k}\left(1+\frac{1}{z}\right)+(\rho)^2\left(1+\frac{1}{z}\right)} \\
\gamma_3 &= \frac{(\rho)^2\left(1+\frac{1}{z}\right)}{\frac{1}{k}\left(1+\frac{1}{z}\right)+(\rho)^2\left(1+\frac{1}{z}\right)} \\
\gamma_4 &= \frac{z\left(1+kf(\theta)\right)}{k+z\left(1+kf(\theta)\right)} \\
\gamma_5 &= \frac{\frac{1}{(\rho)^2}\frac{1}{k}\left(1+\frac{1}{z}\right)}{1+\frac{1}{z}+\frac{1}{(\rho)^2}\frac{1}{k}\left(1+\frac{1}{z}\right)}
\end{aligned}$$

and :

$$f(\theta) = \frac{1-k+k\left(\frac{1}{2k^2\theta^2}\left(-1-k\theta^2+2k^2\theta^2+\sqrt{\left(1+k\theta^2-2k\theta\right)\left(k\theta^2+2k\theta+1\right)}\right)\right)^2}{1-(k\theta)^2\left(1-\frac{1}{2k^2\theta^2}\left(-1-k\theta^2+2k^2\theta^2+\sqrt{\left(1+k\theta^2-2k\theta\right)\left(k\theta^2+2k\theta+1\right)}\right)\right)^2}\theta^2$$

The numerical study of this set of equation plus the condition $\beta\Delta L_t = 0$ yields the curves on figure 2, drawn for two values of $\kappa = 1, \kappa = 2$.

The case $\kappa \to \infty$ can be obtained by pushing the curves on the Horizontal axis, that is a situation where the transparency is always optimal.

The intercept with the z axis can be found analytically. Actually, the case, $k \to 0$ yields the following result : at the first order one has $y = (1-k)\,\theta^2$

$x = k\theta^2$, $\rho = 1 + \frac{z-1}{z}k$, $\gamma_1 = 1 - \frac{k}{1+\frac{1}{z}}$, $\gamma_2 = \frac{1+kz}{1+z}$, $\gamma_3 = k$, $\gamma_4 = 1 - \frac{k}{z}$, $\gamma_0 = k\theta^2$, $\gamma_5 = 1-k$, $\theta = -z+\left(-2z+1-z^2-\kappa z^2-\kappa z\right)k$, $\tau = \frac{(z-1)\left(1+\varsigma^2\right)k}{\varsigma^2(1-\varsigma^2)}$ and $\beta\Delta L_t = \frac{z-2\kappa-4}{z+1}$ so that transparency dominates when $z > 2\kappa + 4$

Proof of Creative Opacity

A difference with the two periods model is that here, we in fact compare the losses over only one period. For opacity to welfare-dominate transparency, therefore, it must be that it reduces inflation volatility in period t by enough to offset the welfare loss of period 2. We know that period 1 inflation is driven by expected inflation and the extent to which the central bank fails to stabilize output in period 1.

$$\begin{aligned}
\pi_t &= -(2+\kappa)\kappa E_t^P r_{t+1} - \kappa r_t + \varepsilon_t + (1+\kappa) E_t^P \varepsilon_{t+1} - \kappa E_t^P \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} r_{t+n} \\
&= (2+\kappa) E_t^P (\varepsilon_{t+1} - \kappa r_{t+1}) - \kappa E_t^P \sum_{n=2}^{\infty} \frac{1-\varsigma^{2n+2}}{\varsigma^n (1-\varsigma^2)} r_{t+n} - \psi
\end{aligned}$$

with

$$\begin{aligned}
\psi &= \kappa r_t - (\varepsilon_t - E_t^P \varepsilon_{t+1}) \\
&= -\varepsilon_{t,t+1}^{CB} + \rho \varepsilon_{t,t}^{CB} + (1-\rho) s_t + \left(\gamma_2 \varepsilon_{t,t+1}^P + (1-\gamma_2)\left(\varepsilon_{t,t+1}^{CB} - \rho\left(\varepsilon_{t,t}^{CB} - \varepsilon_{t,t}^P\right)\right) - \varepsilon_t\right) \\
&= -\gamma_2 \varepsilon_{t,t+1}^{CB} + \rho\gamma_2\left(\varepsilon_{t,t}^{CB} - \varepsilon_t\right) + (1-\rho)(s_t - \varepsilon_t) + (1-\gamma_2)\rho\left(\varepsilon_{t,t}^P - \varepsilon_t\right) + \gamma_2 \varepsilon_{t,t+1}^P
\end{aligned}$$

The "policy miss" term measures the private sector's perception of the extent to which the central bank fails to achieve its period 1 objective when it optimally chooses r_t. Actually, $\pi_t + \psi$ measures rather to what extent the private sector trusts or not the central bank to fight efficiently inflation in the future.

ψcan be written explicitely in both regimes. In transparency:

$$\beta Var^{tr}(\psi) = \frac{1}{z+k} + \frac{1}{kz(z+1)}$$

and in opacity:

$$\begin{aligned}
\beta Var^{op}(\psi) &= \frac{\gamma_2^2}{k}\left(1+\frac{1}{z}\right) + \frac{(\rho\gamma_2)^2}{z} + \frac{((1-\rho)+k(1-\gamma_2)\rho)^2}{k} \\
&\quad + (1-k)((1-\gamma_2)\rho)^2 + (1-\rho)^2 f(\theta)
\end{aligned}$$

Under transparency, but not under opacity $E_t^P(\varepsilon_{t+1} - r_{t+1}) = E_t^P \pi_{t+1} = 0$ (due to the alignment of expectations $E_t^P \varepsilon_{t+1} = E_t^P\ E_t^{CB} \varepsilon_{t+1}$), so opacity tends to add volatility to period t inflation with $Var^{op}(\varepsilon_{t+1} - \kappa r_{t+1})$, the more so the higher is κ. Furthermore, we can show numerically that $Var^{op}(\psi) > Var^{tr}(\psi)$ for the solutions of the model.

It follows that opacity always raises period t inflation as well, unless $cov(E_t^P(\varepsilon_{t+1} - \kappa r_{t+1}), \psi)$ under opacity is positive. Thus $cov(E_t^P(\varepsilon_{t+1} - \kappa r_{t+1}), \psi) > 0$ is a necessary and sufficient, condition for opacity to raise welfare. The graphical analysis shows that his is the case under the curve in Figure 1.

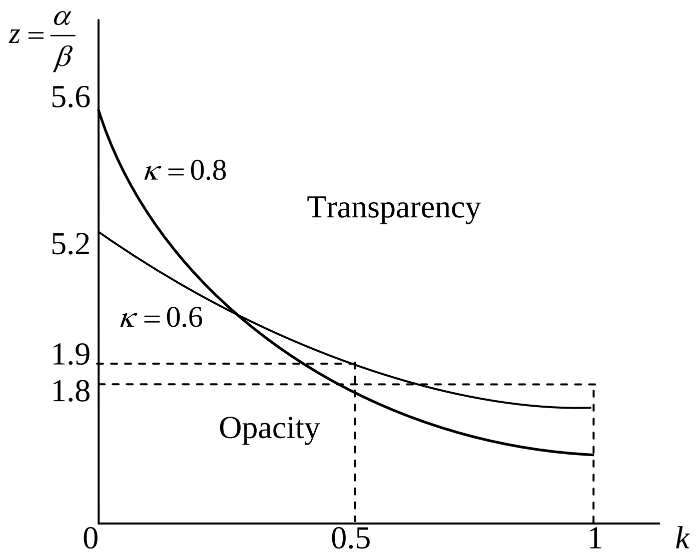
$z = \frac{\alpha}{\beta}$
5.6
$\kappa = 0.8$
Transparency
5.2
$\kappa = 0.6$
1.9
1.8
Opacity
0
0.5
1
k

Figure 1