

# The Stimulative Effect of Forward Guidance\*

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## Abstract

This paper examines the stimulative effect of central bank forward guidance—the promise to keep future policy rates lower than its policy rule suggests—when the short-term nominal interest rate is stuck at its zero lower bound (ZLB). We utilize a standard New Keynesian model in which forward guidance enters our model as news shocks to the monetary policy rule. Three key findings emerge: (1) Forward guidance is more stimulative at the ZLB when households believe the economic recovery will be strong. When households expect a weak recovery or initially have low confidence in the economy, forward guidance is less stimulative because interest rates are already expected to remain low; (2) Longer forward guidance horizons do not cause the stimulative effect to explode or reverse, but rather spread the effect across the entire horizon; and (3) Failing to include a ZLB constraint causes the model to substantially overstate the stimulative effect of forward guidance. Given those findings, we use Blue Chip survey data to compare our model’s predictions of the stimulative effect of forward guidance to data before and after the Fed’s historic policy announcement on August 9, 2011. The results in that case study provide an explanation for the Forward Guidance Puzzle—the claim that New Keynesian models overestimate the effect of forward guidance.

*Keywords:* Monetary Policy; Forward Guidance; Zero Lower Bound; Nonlinear Solution

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# 1 INTRODUCTION

This paper studies the effect of central bank forward guidance when the nominal interest rate is stuck at its zero lower bound (ZLB). Forward guidance refers to central bank communication about future policy, which takes many forms including announcements about objectives, contingencies, and policy actions. The decision of central banks to announce their expected policy rate path is a recent phenomenon that has accompanied the evolution of inflation targeting.

Campbell et al. (2012) introduce terms to differentiate the two types of forward guidance: Delphic and Odyssean. Delphic forward guidance is a central bank's forecast of its own policy, which is based on its projections for inflation and real activity as well as an established policy rule. Odyssean forward guidance is a commitment to deviate from the established policy rule at some point in the future when the policy rate is normally expected to rise above zero. The commitment usually is to keep the interest rate at zero longer than the policy rule suggests. Typically, forward guidance about the policy rate path is consistent with the expected state of the economy and an established policy rule (Delphic forward guidance). If the policy rule is known, then Delphic forward guidance on the policy rate path is redundant. One reason to announce forward guidance about the policy rate together with economic forecasts is to clarify the central bank's policy strategy.<sup>1</sup>

Central banks utilize Odyssean forward guidance when the short-term nominal interest rate is constrained at its ZLB because they no longer have the ability to stimulate demand with traditional open market operations. Several papers, including this one, argue that Odyssean forward guidance can raise output when the policy rate is at the ZLB.<sup>2</sup> To stimulate the economy, the central bank commits to a policy rate path that is lower than the interest rate path it would be expected to follow in the absence of forward guidance. We assume households understand the policymaker's objective and do not interpret the looser policy as a signal that the central bank is more pessimistic about the recovery. The forward guidance applies to future periods in which households expect a recovery to raise the policy rate. The promise to keep the policy rate low during this part of the recovery generates a reduction in the expected policy rate. By lowering the expected future policy rate, forward guidance also reduces current long-term nominal interest rates (through the expectations theory of the term structure) which raises demand in the near term.

We calculate the global solution to a nonlinear New Keynesian model to examine the stimulative effect of forward guidance when the ZLB binds. Forward guidance enters our model as news shocks to the monetary policy rule similar to Laséen and Svensson (2011) which commits the central bank to a lower policy rate than its policy rule suggests. Although many papers employ nonlinear methods for examining the effects of a ZLB constraint,<sup>3</sup> this paper is the first to utilize nonlinear methods to study the stimulative effect of forward guidance at the ZLB.

Some researchers who study forward guidance do not impose the ZLB constraint, but instead, assume the central bank credibly provides forward guidance that it will maintain a low nominal interest rate in the future [Laséen and Svensson (2011) and Del Negro et al. (2012)]. Although

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<sup>1</sup>See the Bank of England (2013) for a discussion of how forward guidance helps the public form more accurate expectations about future central bank policy. For a collection of essays about forward guidance by central bankers, market participants, and academics, see den Haan (2013). For a detailed account of recent unconventional monetary policies used by central banks, including forward guidance, see International Monetary Fund (2013).

<sup>2</sup>See Krugman (1998), Reifschneider and Williams (2000), Eggertsson and Woodford (2003), Nakov (2008), Walsh (2009), Levin et al. (2010), Werning (2011), Campbell et al. (2012), Del Negro et al. (2012), and Woodford (2012).

<sup>3</sup>See, for example, Wolman (2005), Basu and Bundick (2012), Fernández-Villaverde et al. (2012), Nakata (2012), Aruoba and Schorfheide (2013), Gust et al. (2013), Mertens and Ravn (2013), and Gavin et al. (2014).

that assumption makes the model more numerically tractable, it means that households can expect negative nominal interest rates. That fact is particularly important when studying forward guidance because households will believe unanticipated expansionary monetary policy shocks are stimulative even when the ZLB binds. It also indicates that when households receive news about a future reduction in the nominal interest rate, they will expect negative future nominal interest rates. Those facts, together, will cause the model to overstate the stimulative effect of forward guidance. In addition, the problem of accurately capturing households' expectations is exacerbated when the forward guidance horizon spans multiple years. We focus on the impact of forward guidance when the nominal interest rate is at the ZLB, but also show its effects when the ZLB does not bind.

Three key findings emerge from our analysis of Odyssean forward guidance:

1. Forward guidance is more stimulative at the ZLB when households believe the economic recovery will be strong. Those optimistic feelings raise next period's expected nominal interest rate. The central bank's promise to provide a future expansionary monetary policy shock, however, dampens that increase in the expected interest rate, which boosts current consumption. Conversely, the expectation of a sluggish recovery results in forward guidance having a smaller stimulative effect because interest rates are already expected to remain low.
2. Longer forward guidance horizons do not reverse the stimulative effect or cause it to explode, but instead, spread the effect across the horizon by flattening the yield curve. Laséen and Svensson (2011) and Carlstrom et al. (2012) show that the qualitative effects of key economic variables to forward guidance can change depending on how long the central bank promises to maintain a low policy rate. We examine horizons up to 10 quarters and find no evidence that forward guidance generates reversals such as deflation or output losses.
3. The failure to incorporate a binding ZLB constraint into the model causes the stimulative effect of forward guidance to be much more pronounced. That overestimation occurs because the absence of a ZLB constraint enables additional stimulus to be generated by pushing the current and expected nominal interest rates below zero.

Given those findings, we use Blue Chip survey data to compare our model's predictions to economic forecasts before and after the Fed's historic forward guidance announcement on August 9, 2011. Our results in that case provide an explanation for the Forward Guidance Puzzle—the claim that New Keynesian models overestimate the effect of forward guidance [Del Negro et al. (2012)].

Many papers that study forward guidance assume the central bank pledges to maintain a particular interest rate peg for a specified period of time [Laséen and Svensson (2011), Carlstrom et al. (2012), and Del Negro et al. (2012)]. The effect of forward guidance on the nominal interest rate is measured as the difference between the interest rate peg and the interest rate that would occur in the absence of forward guidance. To preserve the peg, the central bank must provide news over the entire forward guidance horizon to offset shocks that affect the interest rate. Thus, there are two dimensions to a forward guidance policy: its horizon and the intensity of the news (i.e., the variance of the news process). Del Negro et al. (2012) show that when they extend the forward guidance horizon, it leads to predictions that overstate the increase in output and inflation. Their experiment, however, relies on a stronger intensity of news to maintain the interest rate peg in the future. In our analysis, we weight the news shocks to keep the variance of the news process constant, which allows us to isolate the effects of a longer forward guidance horizon from a change in the intensity of news. Under that assumption, we find that longer forward guidance horizons generate higher consumption and inflation at a decreasing rate when the policy is implemented.

Our theoretical analysis of forward guidance builds on the seminal work of Eggertsson and Woodford (2003), who study how to conduct monetary policy when the nominal interest rate is at the ZLB. They focus on an announced price level path target, rather than forward guidance about future policy rates. Although their analysis is thorough, it is based on a linearized version of a constrained New Keynesian model in which there is no expectation of returning to the ZLB after exiting. The limitations of those features, which are discussed in Braun et al. (2012), Fernández-Villaverde et al. (2012), and Gavin et al. (2014), motivate us to solve a fully nonlinear model with endogenous ZLB events to more accurately account for the expectational effects of the ZLB.

The paper is organized as follows. [Section 2](#) provides a post-financial crisis account of the Fed's forward guidance in its policy statements. [Section 3](#) describes the nonlinear model, including the specification of forward guidance, its calibration, and its solution method. [Section 4](#) quantifies the effect of forward guidance. Specifically, we describe the decision rules and the impulse responses to news shocks across alternative forward guidance horizons. [Section 5](#) focuses on the Federal Open Market Committee (FOMC)'s August 9, 2011 forward guidance announcement in order to examine how our model's predictions compare with the data. [Section 6](#) concludes.

## 2 FORWARD GUIDANCE AT THE FEDERAL RESERVE

The FOMC uses two methods to communicate information about the path of future policy rates. One, it releases the individual forecasts of its participant members four times per year, but that information can be diverse and reveals differences in opinions. Two, it provides forward guidance about the future federal funds rate in its policy statements and has consistently done so since 2008.

At the December 16, 2008 meeting, the FOMC lowered the federal funds rate to a range of 0-0.25 percent and announced the rate would remain at that unusually low level for an *extended* period. The FOMC continued to use that vague language until its August 9, 2011 meeting in which it stated the low range was likely warranted "... *at least through mid-2013.*" That announcement was the first time the FOMC used date-based forward guidance, and it had a dramatic effect on current and expected future interest rates. Using a distribution of expected future Eurodollar rates, Raskin (2013) estimates that the August 9, 2011 statement generated both a significant decline in 1- to 5-year ahead future Eurodollar rates and a substantial reduction in the dispersion of those expected interest rates. For example, the 2-year ahead Eurodollar rate fell by about 25 basis points after the statement was released. Moessner (2013) also finds that the 1-year ahead U.S. Treasury bill rate at 3- to 8-year horizons fell by about the same amount.

The next change in forward guidance occurs in the statement released after the January 25, 2012 FOMC meeting. That policy statement was different in two ways. One, the time that the federal funds rate was expected to remain at zero was updated to read "... *at least through late 2014,*" which was an increase of six quarters. Two, the FOMC expressed a more pessimistic economic outlook and indicated the projected path for the federal funds rate was conditional on that outlook. Subsequent speeches by policymakers provided support for that assessment.

We contend that the forward guidance provided in the January 25, 2012 statement is Delphic for two reasons. One, the statement expressed more pessimism about the economy, which suggests the FOMC's policy rule was projecting a later date for raising the federal funds rate. Two, the FOMC never stated the new projected interest rate path was different from the path implied by its policy rule. When analyzing the data, Raskin (2013) finds the 6-quarter extension of forward guidance marginally reduced expected interest rates and was not statistically significant. He argues

that the difference in the market's reaction to the August 9, 2011 and the January 25, 2012 events is attributed to the beliefs of market participants. Specifically, the market was surprised by the first announcement of calendar-based guidance, but not by the second statement of a longer forward guidance horizon. He also suggests that the greater uncertainty about the longer horizon may have made the later announcement less relevant for current expectations.

By late summer 2012, the economy continued to disappoint policymakers and the statement issued following the September 13, 2012 meeting was amended to read:

To support continued progress toward maximum employment and price stability, . . . *a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens*. . . the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be *warranted at least through mid-2015*.

That statement included a 2-quarter extension to the time the FOMC promised to maintain a zero federal funds rate and a new pledge to add \$85 billion to the Fed's balance sheet every month until the labor market significantly improved. The language ". . . for a considerable time after the economic recovery strengthens" conveys Odyssean forward guidance. On the other hand, the FOMC likely dampened economic expectations because the statement expressed more pessimism about business spending, and FOMC participants lowered their GDP growth forecasts.

On December 12, 2012, the FOMC again adjusted its forward guidance from the calendar-based language "at least through mid-2015" to forward guidance based on unemployment and expected inflation. The policy statement read:

. . . this exceptionally low range for the federal funds rate will be appropriate *at least as long as the unemployment rate remains above 6-1/2 percent*, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

The FOMC participants' forecasts released after the meeting indicate that the unemployment rate would likely hit 6-1/2 percent in mid-2015. Therefore, that statement was not designed to change expectations about when the policy rate would rise, but only to emphasize that the timing of the FOMC's decision to increase its policy rate is conditional on inflation expectations and the state of the labor market. On December 18, 2013, the policy statement sought to reinforce the Odyssean nature of forward guidance by stating:

. . . it likely will be appropriate to maintain the current target range for the federal funds rate *well past the time that the unemployment rate declines below 6-1/2 percent*, especially if projected inflation continues to run below the Committee's 2 percent longer-run goal.

Evidence shows the labor market had improved and as a result, the FOMC decided to begin tapering their monthly asset purchases. The new language on unemployment was probably added to prevent the market from moving up the date in which they expect the federal funds rate to rise.

Market and media reaction over the past few years to the Fed's policy statements indicate the FOMC has had limited success in communicating Odyssean forward guidance.<sup>4</sup> This paper highlights the conditions under which Odyssean forward guidance is stimulative.

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<sup>4</sup>The Bank of Japan, the European Central Bank, and the Bank of England have also communicated Odyssean forward guidance. See Filardo and Hofmann (2014) for an overview of the policies and their economic impacts. Norway, Sweden, and New Zealand, however, have relied more on Delphic forward guidance. Their policies are examined by Andersson and Hofmann (2010) and Kool and Thornton (2012).

### 3 ECONOMIC MODEL, CALIBRATION, AND SOLUTION METHOD

We use a conventional New Keynesian model without capital in which the ZLB on the nominal interest rate occasionally binds due to persistent discount factor shocks. Our innovation is to introduce Odyssean forward guidance on the nominal interest rate via monetary policy shocks that households observe before they hit the economy while also imposing the ZLB.

**3.1 HOUSEHOLDS** A unit measure of identical households choose  $\{c_t, n_t, b_t\}_{t=0}^{\infty}$  to maximize expected lifetime utility,  $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log c_t - \chi n_t^{1+\eta}/(1+\eta)]$ , where  $1/\eta$  is the Frisch elasticity of labor supply,  $c_t$  is consumption of the final good,  $n_t$  is labor hours,  $E_0$  is an expectations operator conditional on information available in period 0,  $\tilde{\beta}_0 \equiv 1$ , and  $\tilde{\beta}_t = \prod_{i=1}^t \beta_i$  for  $t > 0$ .  $\beta_i$  is a time-varying subjective discount factor that evolves according to

$$\beta_i = \bar{\beta}(\beta_{i-1}/\bar{\beta})^{\rho_\beta} \exp(v_i), \quad (1)$$

where  $\bar{\beta}$  is the steady-state discount factor,  $0 \leq \rho_\beta < 1$ , and  $v_i \sim \mathbb{N}(0, \sigma_v^2)$ . Those choices are constrained by  $c_t + b_t = w_t n_t + r_{t-1} b_{t-1}/\pi_t + d_t$ , where  $\pi_t = p_t/p_{t-1}$  is the gross inflation rate,  $w_t$  is the real wage rate,  $b_t$  is a 1-period real bond,  $r_t$  is the gross nominal interest rate, and  $d_t$  are profits from intermediate firms. The optimality conditions to each household's problem imply

$$w_t = \chi n_t^\eta c_t, \quad (2)$$

$$1 = r_t E_t [\beta_{t+1} (c_t/c_{t+1})/\pi_{t+1}]. \quad (3)$$

**3.2 FIRMS** The production sector consists of monopolistically competitive intermediate goods firms who produce a continuum of differentiated inputs and a representative final goods firm. Each firm  $i \in [0, 1]$  in the intermediate goods sector produces a differentiated good,  $y_t(i)$ , with identical technologies given by  $y_t(i) = n_t(i)$ , where  $n_t(i)$  is the level of employment used by firm  $i$ . Each intermediate firm chooses its labor supply to minimize operating costs,  $w_t n_t(i)$ , subject to its production function. Using a Dixit and Stiglitz (1977) aggregator, the representative final goods firm purchases  $y_t(i)$  units from each intermediate goods firm to produce the final good,  $y_t \equiv [\int_0^1 y_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$ , where  $\theta > 1$  measures the elasticity of substitution between the intermediate goods. Profit maximization then yields the demand function for intermediate inputs given by  $y_t(i) = (p_t(i)/p_t)^{-\theta} y_t$ , where  $p_t = [\int_0^1 p_t(i)^{1-\theta} di]^{1/(1-\theta)}$  is the price of the final good.

Following Rotemberg (1982), each firm faces a cost to adjusting its price,  $adj_t(i)$ , which emphasizes the negative effect that price changes can have on customer-firm relationships. Using the functional form in Ireland (1997),  $adj_t(i) = \varphi [p_t(i)/(\bar{\pi} p_{t-1}(i)) - 1]^2 y_t/2$ , the real profits of firm  $i$  are  $d_t(i) = [(p_t(i)/p_t) y_t(i) - (w_t n_t(i) + adj_t(i))]$ , where  $\varphi \geq 0$  controls the size of the adjustment cost and  $\bar{\pi}$  is the steady-state gross inflation rate. Each intermediate goods firm chooses its price,  $p_t(i)$ , to maximize the expected discounted present value of real profits  $E_t \sum_{k=t}^{\infty} \lambda_{t,k} d_k(i)$ , where  $\lambda_{t,t} \equiv 1$ ,  $\lambda_{t,t+1} = \beta_{t+1} (c_t/c_{t+1})^\sigma$  is the stochastic pricing kernel between periods  $t$  and  $t+1$  and  $\lambda_{t,k} \equiv \prod_{j=t+1}^k \lambda_{j-1,j}$ . In a symmetric equilibrium, all intermediate goods firms make the same decisions and the optimality condition reduces to

$$\varphi \left( \frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} = (1 - \theta) + \theta w_t + \varphi E_t \left[ \lambda_{t,t+1} \left( \frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{y_{t+1}}{y_t} \right]. \quad (4)$$

In the absence of price adjustment costs (i.e.,  $\varphi = 0$ ), the real wage rate equals  $(\theta - 1)/\theta$ , which is the inverse of a firm's markup of price over marginal cost.



**3.3 MONETARY POLICY** Households receive forward guidance (news) about future monetary policy through discretionary monetary policy shocks. The households believe the central bank is fully credible and will keep its commitments on future policy rates. Specifically, the central bank sets the gross nominal interest rate according to the following Taylor rule:

$$r_t = \max\{1, \bar{r}(\pi_t/\pi^*)^{\phi_\pi} \exp(x_t)\},$$

$$x_t \equiv \sum_{j=0}^q \alpha_j \varepsilon_{t-j}, \quad \sum_{j=0}^q \alpha_j^2 = 1, \quad (5)$$

where  $\pi^*$  is the gross inflation rate target,  $\phi_\pi$  is the policy response to inflation,  $\varepsilon_i \sim i.i.d. \mathbb{N}(0, \sigma_\varepsilon^2)$  is a monetary policy shock,  $\alpha_j$  is the intensity of the news  $j$  periods in the future, and  $q > 0$  is the forward guidance horizon. For example, when  $(\alpha_0, \alpha_1, \dots, \alpha_q) = (1, 0, \dots, 0)$ , the shock is unanticipated (no forward guidance) and when  $(\alpha_0, \alpha_1, \dots, \alpha_q) = (0, 0, \dots, 1)$ , households anticipate the shock  $q$  periods before it occurs ( $q$ -period forward guidance). The restriction on  $\alpha_j$  guarantees that the variance of  $x_t$  is the same as  $\varepsilon_t$ . In other words, the distribution of the news does not affect the variance of the monetary policy shock process. That restriction is particularly important because it isolates the effect of lengthening the forward guidance horizon. In general, the share of news that households receive  $j$  periods ahead equals  $\alpha_j / \sum_{i=0}^q \alpha_i$ .

**3.4 EQUILIBRIUM, CALIBRATION, AND SOLUTION METHOD** The resource constraint is given by  $c_t = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2]y_t = \tilde{y}_t$ , where  $\tilde{y}_t$  includes the value added by intermediate firms, which is their output minus quadratic price adjustment costs. A competitive equilibrium consists of sequences of quantities  $\{c_t, n_t, y_t, b_t\}_{t=0}^\infty$ , prices  $\{w_t, r_t, \pi_t\}_{t=0}^\infty$ , and discount factors  $\{\beta_t\}_{t=0}^\infty$  that satisfy the exogenous driving process for the discount factor, (1), each household's and each firm's optimality conditions, (2)-(4), the monetary policy rule, (5), the production function,  $y_t = \int_0^1 n_t(i) = n_t$ , the bond market clearing condition,  $b_t = 0$ , and the resource constraint.

We calibrate the model at a quarterly frequency using values common in the monetary policy literature. The risk-free real interest rate is set to 4 percent annually, which implies a steady-state quarterly discount factor,  $\bar{\beta}$ , equal to 0.99. The Frisch elasticity of labor supply,  $1/\eta$ , is set to 1 and the leisure preference parameter,  $\chi$ , is set so that steady-state labor equals  $1/3$  of the available time. The price elasticity of demand between intermediate goods,  $\theta$ , is calibrated to 6, which corresponds to an average markup of price over the real wage rate equal to 20 percent. The costly price adjustment parameter,  $\varphi$ , is set to 58.25, which is similar to a Calvo (1983) price-setting specification in which prices change on average once every four quarters ( $\omega = 0.75$ ).<sup>5</sup> In the policy sector, the steady-state gross inflation rate,  $\bar{\pi}$ , is calibrated to 1.005 so that the annual inflation rate target is 2 percent. We set the monetary response to changes in inflation,  $\phi_\pi$ , equal to 1.5. A determinate minimum state variable (MSV) solution requires that  $\rho_\beta$  and  $\sigma_v$  are not too large. We set  $\rho_\beta = 0.8$  and  $\sigma_v = 0.0025$  which yields a convergent MSV solution [Richter and Throckmorton (2013)]. Those are also the same values used in Fernández-Villaverde et al. (2012). The standard deviation of the monetary policy shock,  $\sigma_\varepsilon$ , is set equal to 0.0025 (25 basis points).

We solve the model using the policy function iteration algorithm described in Richter et al. (2013), which is based on the theoretical work on monotone operators in Coleman (1991). This

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<sup>5</sup>If  $\omega$  represents the fraction of firms that cannot adjust prices each period, then  $\varphi = \omega(\theta - 1)/[(1 - \omega)(1 - \beta\omega)]$  in a linear model with a zero-inflation steady state, which provides a reasonable estimate of the adjustment costs parameter.

solution method discretizes the state space and uses time iteration to solve for the updated policy functions until the tolerance criterion is met. We use piecewise linear interpolation to approximate future variables that show up in expectations, since this approach more accurately captures the kink in the policy functions than continuous functions, and Gauss-Hermite quadrature to numerically integrate. Those techniques capture the expectational effects of going to and returning to the ZLB. For a more formal description of the numerical algorithm and convergence, see [Appendix A](#).

## 4 DECISION RULES, IMPULSE RESPONSES, AND FORWARD GUIDANCE

**4.1 ONE-QUARTER HORIZON ( $q = 1$ )** [Figure 1](#) plots the decision rules for consumption and the current and expected future nominal interest rate as a function of the current,  $\hat{\epsilon}_t$ , and lagged,  $\hat{\epsilon}_{t-1}$ , monetary policy shocks.<sup>6</sup> We highlight the economic effects of monetary policy shocks,  $\hat{\epsilon}_{t-1}$  and  $\hat{\epsilon}_t$ , ranging from  $-1.2$  to  $1.2$  in a model with and without forward guidance. The subscript on the shock represents the period in which households learn about it and not necessarily the period the shock impacts the economy. If, for example, the central bank provides no forward guidance, then  $\hat{\epsilon}_t$  represents an unanticipated monetary policy shock, which is observed and impacts the economy in period  $t$ . When the central bank provides 1-quarter forward guidance,  $\hat{\epsilon}_{t-1}$  represents a news shock that households learn about in period  $t - 1$  but does not impact the economy until period  $t$ . We assume  $\hat{\beta}_t = 1.15$  in all our experiments unless otherwise specified. In the absence of any news shock,  $1.15$  is the minimum value of  $\hat{\beta}_t$  at which the ZLB binds in period  $t$ . The high discount factor initially depresses consumption by about 1.3 percent. Households, however, expect that the discount factor will follow its law of motion and revert to its steady state over time. Assuming the central bank does not intervene, that belief raises the expected nominal interest rate for next period and all subsequent periods above the ZLB (i.e.,  $E_t(r_{t+i}) > 0$  for all  $i = 1, 2, \dots, \infty$ ). For example, next period's expected nominal interest rate increases by 0.35 percentage points.

When  $(\alpha_0, \alpha_1) = (1, 0)$  (solid line), the central bank provides no forward guidance, so  $\hat{\epsilon}_{t-1}$  represents an unexpected shock last period that has no effect on the current or expected future nominal interest rate. Therefore, the decision rules for the no forward guidance case are flat with respect to  $\hat{\epsilon}_{t-1}$ . In contrast, an unexpected shock in the current period,  $\hat{\epsilon}_t$ , may have an effect on the economy, depending on its sign. When  $\hat{\epsilon}_t > 0$ , the central bank conducts a contractionary policy, which causes the current nominal interest rate to rise and lowers current consumption. Next period's expected nominal interest rate is unaffected since the shock is serially uncorrelated. If, on the other hand,  $\hat{\epsilon}_t < 0$ , then monetary policy is expansionary, but it has no impact on the current nominal interest rate since it is already at the ZLB. Thus, the decision rules are flat when  $\hat{\epsilon}_t < 0$ .

When  $(\alpha_0, \alpha_1) = (0, 1)$  (dashed line), the central bank provides households with 1-quarter forward guidance. Suppose they receive news in period  $t - 1$  of an expansionary monetary policy shock,  $\hat{\epsilon}_{t-1} < 0$ , that impacts the policy rule in period  $t$ . That pledge to lower the nominal interest rate has no effect on the economy in period  $t$  because the nominal rate is already at its ZLB.<sup>7</sup>

A central bank announcement this period about future monetary policy shocks can affect the current economy even though the shock has not yet happened. Suppose, for example, households receive information in period  $t$  that an expansionary monetary policy shock,  $\hat{\epsilon}_t < 0$ , will occur in

<sup>6</sup>In all our results, a hat denotes percent deviation from the deterministic steady state (i.e., for some generic variable  $x$  in levels,  $\hat{x}_t \equiv 100((x_t - \bar{x})/\bar{x})$  and a tilde denotes a net rate (i.e., for some gross rate  $x$ ,  $\tilde{x}_t = 100(x_t - 1)$ ).

<sup>7</sup>As noted above, Odyssean forward guidance seeks to lower interest rates by providing news on current and future expansionary monetary policy shocks. Therefore, we focus on examining the impact of expansionary news shocks.



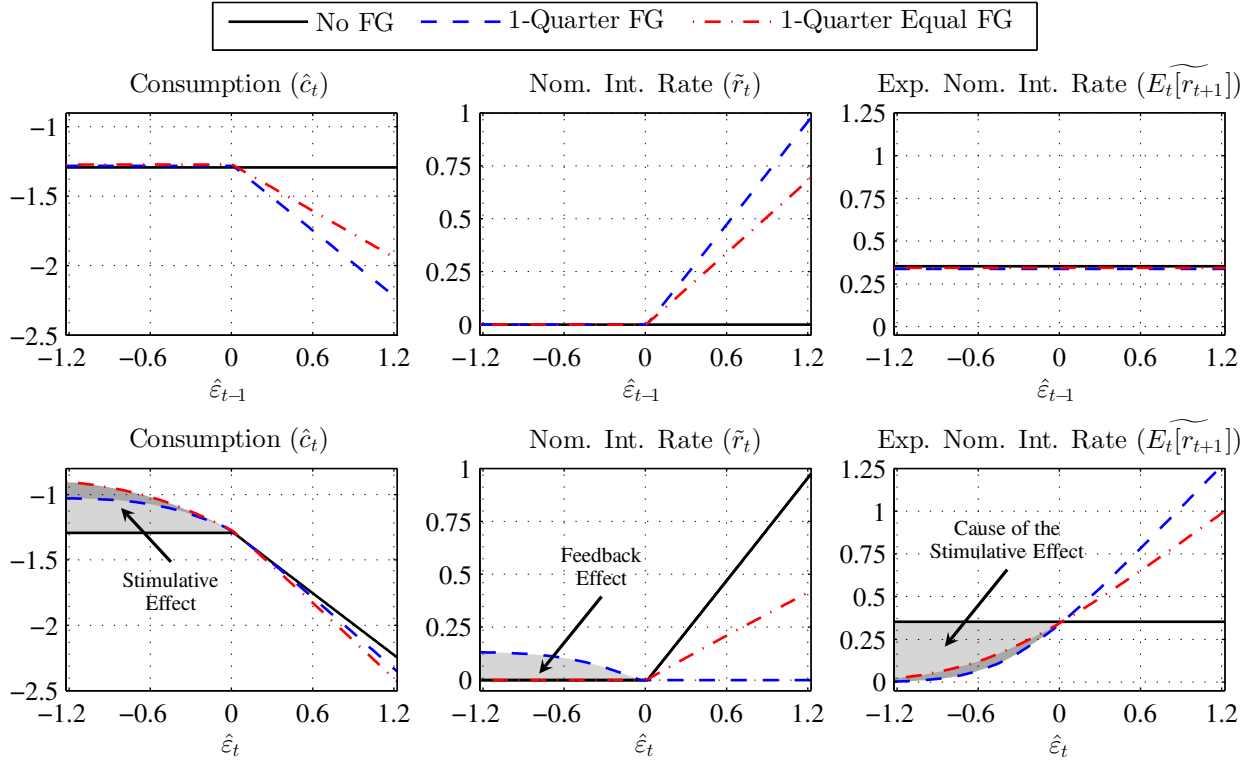


Figure 1: Consumption and nominal interest rate decision rules as a function of the current and lagged monetary policy shocks with no forward guidance,  $(\alpha_0, \alpha_1) = (1, 0)$  (solid line), 1-quarter forward guidance,  $(\alpha_0, \alpha_1) = (0, 1)$  (dashed line), and 1-quarter equal forward guidance,  $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$  (dash-dotted line). The decision rules are based on a cross section of the state space where  $\hat{\beta}_t = 1.15$ , which is the minimum value of the discount factor necessary for the ZLB to bind. The horizontal axis displays the news/policy shock, which is specified in percentages. The vertical axis shows the values for consumption, which are in percent deviations from its stochastic steady state and the current and expected nominal interest rate, which are in net percentages.

period  $t+1$ . If the discount factor is *not* expected to revert to its mean (i.e., the economy is expected to remain stagnant), then forward guidance will have little economic effect since households expect that next period's nominal interest rate will remain near its ZLB. In our setup, the discount factor,  $\hat{\beta}_t = 1.15$ , is above its mean and households expect it to decline in the future. That expectation pushes up expected nominal interest rates. When households are informed this period about an expansionary monetary policy shock next period, they expect next period's nominal interest rate to increase less than if they received no forward guidance. That expectational effect stimulates current consumption and raises inflation, even though the discount factor remains at  $\hat{\beta}_t = 1.15$ .

Another way to understand how forward guidance stimulates current consumption is through an intertemporal consumption smoothing motive. Households know an expansionary monetary policy shock will occur in period  $t+1$  and expect higher future consumption. Consequently, households raise current consumption to smooth their consumption path. That increased consumption pushes up inflation, which feeds into the Taylor rule and drives up the current nominal interest rate. We call that rise in the nominal interest rate the *feedback effect* from forward guidance. The shaded regions in [figure 1](#) represent the effects of forward guidance.

The feedback effect of higher consumption on the nominal interest rate dampens the stimulative effect of 1-quarter forward guidance. Although the central bank could provide *additional* news in

the current period that would mitigate the feedback effect (e.g., set  $(\alpha_0, \alpha_1) = (1, 1)$ ), they could also *redistribute* the news so that the variance of the news process,  $x_t$ , remains constant while still enhancing the stimulative effect. An example of that policy is  $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$  (dash-dotted line), which we call 1-quarter equal forward guidance. That specification equally shocks the policy rule in periods  $t$  and  $t + 1$  but keeps the variance of the news process constant.

The dark-shaded regions in figure 1 represent the marginal effects on the decision rules of switching from 1-quarter forward guidance to 1-quarter equal forward guidance. The effects of 1-quarter equal forward guidance on the decision rules are different than 1-quarter forward guidance in two ways. One, the current expansionary policy shock in 1-quarter equal forward guidance further stimulates the economy because it eliminates the feedback effect that causes the current nominal interest rate to rise with 1-quarter forward guidance. Without that feedback effect, consumption increases more. Two, the smaller expansionary policy shock next period with 1-quarter equal forward guidance moderates both the decline in next period's expected nominal interest rate and the increase in current consumption. Overall, the stimulative effect from the lower current nominal interest rate dominates the negative effects of a smaller drop in next period's expected nominal rate so that consumption rises more with 1-quarter equal forward guidance.

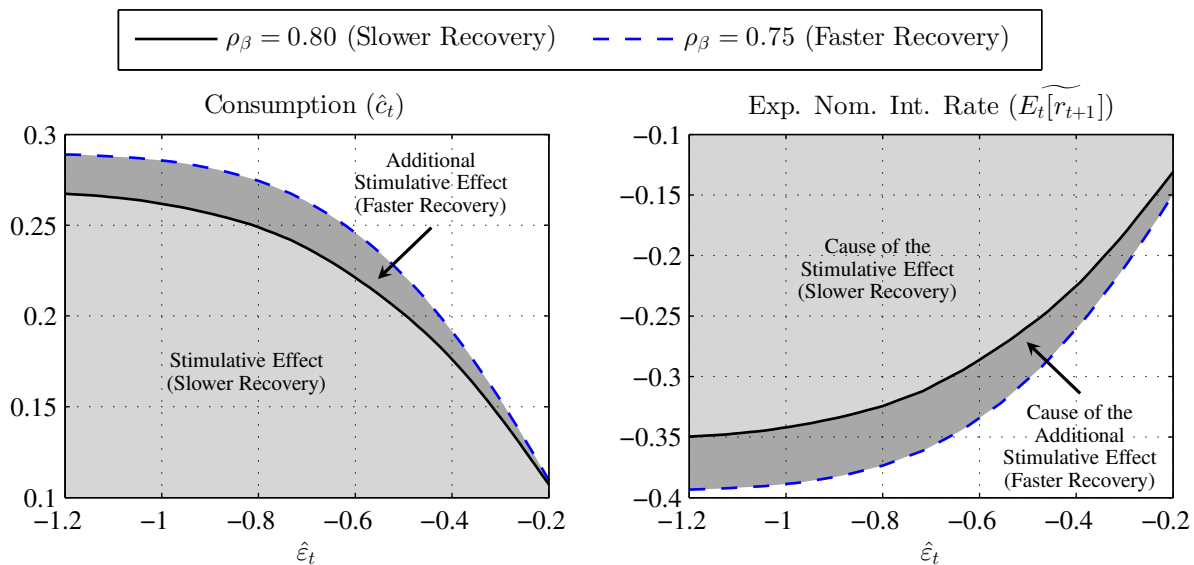


Figure 2: Forward guidance and the strength of the expected recovery. The impact of forward guidance is examined under the assumptions of a slower recovery,  $\rho_\beta = 0.80$  (solid line), and a faster recovery,  $\rho_\beta = 0.75$  (dashed line). The decision rules are based on the assumption that  $\hat{\beta}_t = 1.15$ , which is the minimum value of the discount factor necessary for the ZLB to bind. The horizontal axis displays the news shock, which is specified in percentages. The vertical axis is the difference between 1-quarter forward guidance,  $(\alpha_0, \alpha_1) = (0, 1)$ , and no forward guidance,  $(\alpha_0, \alpha_1) = (1, 0)$ , where consumption is in percent deviations and the nominal interest rate is in percentage points.

Figure 2 shows the stimulative effect of forward guidance is stronger when households expect a faster economic recovery.<sup>8</sup> The left panel displays the decision rules for consumption while the right panel shows the decision rules for next period's expected nominal interest rate. The vertical axis on the left (right) panel illustrates the percentage (percentage point) difference in the decision rules from the model with no forward guidance. The solid line ( $\rho_\beta = 0.80$ ) and the dashed line

<sup>8</sup>Levin et al. (2010) make a similar point in their study of optimal policy in a New Keynesian model with a ZLB.

( $\rho_\beta = 0.75$ ) show the impact  $\rho_\beta$  has on both variables. The discount factor is abnormally high in period  $t$  which depresses current consumption. A lower  $\rho_\beta$  is a proxy for a quicker recovery because the discount rate is expected to return to its steady state faster. A more rapid decline in the discount factor encourages households to raise their expected consumption growth rate, which causes next period's expected nominal interest rate to increase. The larger jump in the expected nominal rate means that a promise by the central bank to maintain a low policy rate in the future will have a greater stimulative effect on the economy.

The light-shaded regions of [figure 2](#) show the benefits of 1-quarter forward guidance for our baseline calibration of  $\rho_\beta = 0.80$ , while the dark-shaded regions display the additional effects from the same forward guidance when  $\rho_\beta = 0.75$ . For example, an announcement this period by the central bank to lower its policy rate by  $-50$  basis points next period causes current consumption to rise by 0.20 percent when  $\rho_\beta = 0.80$  and by 0.22 percent when  $\rho_\beta = 0.75$ . Forward guidance then is more stimulative when households believe the recovery will be stronger. Our finding suggests that if a central bank expresses a more pessimistic economic outlook when communicating information about future monetary policy, then the expectation of a slower recovery will dampen the stimulative effect of forward guidance.

The stimulative effect of forward guidance also depends on households' initial level of confidence. [Figure 3](#) compares the stimulative effect of forward guidance at four alternative cross sections of the discount factor:  $\hat{\beta}_t = 0$  (solid line),  $\hat{\beta}_t = 0.85$  (dashed line),  $\hat{\beta}_t = 1.15$  (circle markers), and  $\hat{\beta}_t = 1.51$  (triangle markers). The discount factor is a proxy for households' confidence level because it determines their willingness to postpone consumption. When the discount factor is high, households prefer to save more, which indicates that they have less confidence in the economy. Like in [figure 2](#), the vertical axis of the left (right) panel in [figure 3](#) shows the percentage (percentage point) difference between the decision rules for 1-quarter forward guidance,  $(\alpha_0, \alpha_1) = (0, 1)$ , and no forward guidance,  $(\alpha_0, \alpha_1) = (1, 0)$ . Thus, the decision rules for the 1-quarter forward guidance case are larger when the vertical axis values are positive while the decision rules for the no forward guidance case are greater when the vertical axis values are negative.

We begin our analysis by examining the case in which the discount factor is at its steady state,  $\hat{\beta}_t = 0$ , which signifies that the current nominal interest rate is far enough from the ZLB that no plausible policy shock will push it to the ZLB. In that situation, an unanticipated expansionary monetary policy shock (i.e., no forward guidance) in the current period generates a larger increase in consumption than a promise to provide the same-sized expansionary monetary policy shock next period (i.e., 1-quarter forward guidance). That finding shows open market operations are more stimulative than forward guidance when the nominal interest rate is far from its ZLB.

The economic effects of an unanticipated expansionary monetary policy shock are more limited when the discount factor is high enough that the policy shock causes the ZLB to bind. Since households expect the economy to improve between this period and next, a cross section exists in which a *promise* to lower future nominal interest rates generates a larger increase in consumption than an equivalent unanticipated expansionary shock to the current nominal interest rate, which cannot fall below zero. Let us consider the cross section where  $\hat{\beta}_t = 0.85$ . A small expansionary policy shock,  $\hat{\varepsilon}_t > -0.5$ , does not drive the current nominal interest rate to its ZLB. A moderate-sized policy shock,  $-1 < \hat{\varepsilon}_t < -0.5$ , however, does push down the current nominal interest rate to its ZLB, but the economic effects of that shock are still stronger than those produced by 1-quarter forward guidance. Finally, a large expansionary policy shock,  $\hat{\varepsilon}_t < -1$ , generates a smaller increase in consumption than an equivalent 1-quarter forward guidance shock.

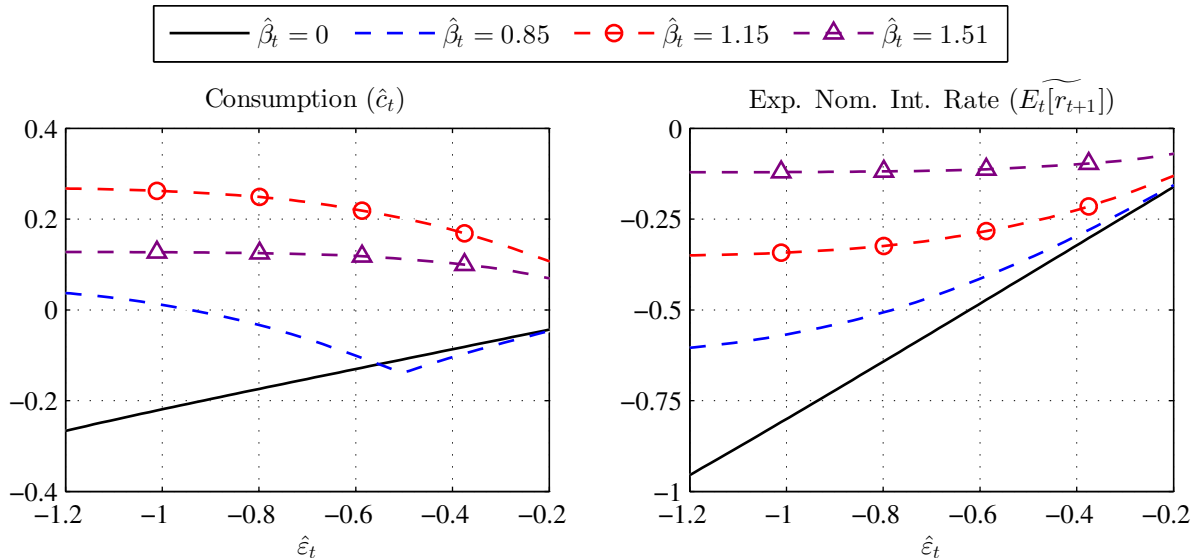


Figure 3: Forward guidance at and away from the ZLB. Four alternative values of the discount factor are considered:  $\hat{\beta}_t = 0$  (solid line),  $\hat{\beta}_t = 0.85$  (dashed line),  $\hat{\beta}_t = 1.15$  (circle markers), and  $\hat{\beta}_t = 1.51$  (triangle markers). The horizontal axis displays the news shock, which is specified in percentages. The vertical axis is the difference between 1-quarter forward guidance,  $(\alpha_0, \alpha_1) = (0, 1)$ , and no forward guidance,  $(\alpha_0, \alpha_1) = (1, 0)$ , where consumption is in percent deviations and the nominal interest rate is in percentage points.

When the ZLB initially binds at  $\hat{\beta}_t = 1.15$ , 1-quarter forward guidance is always more stimulative because an unexpected expansionary monetary policy shock cannot push the nominal rate any lower. As the discount factor gets larger, such as  $\hat{\beta}_t = 1.51$ , households place a smaller probability on exiting the ZLB next period. That lower probability reduces how much households expect next period's nominal interest rate to rise, which limits the economic effects of forward guidance. In our specification, the promise to stimulate the economy next period reduces that period's expected nominal interest rate by a smaller amount as  $\hat{\beta}_t$  rises above 1.15. Those results reinforce our conclusion that a pessimistic economic forecast diminishes the stimulative effect of forward guidance. In fact, the possibility exists that forward guidance can have no stimulative effect at all if the discount factor is sufficiently high. That situation occurs in our model with 1-quarter forward guidance when  $\hat{\beta}_t > 1.85$ . Therefore, it is important to determine the value of the nominal interest rate implied by the Taylor rule in the absence of the ZLB. The more negative that value, the further the economy is from exiting the ZLB and the smaller the stimulative effect of forward guidance. Our finding of a limited stimulative effect of forward guidance at the ZLB offers an alternative explanation for the Forward Guidance Puzzle described in Del Negro et al. (2012).

Another plausible explanation for the Forward Guidance Puzzle is that some models do not impose a ZLB constraint.<sup>9</sup> Suppose, for example, the central bank announces a plan this period to

<sup>9</sup>Del Negro et al. (2012) perform simulations of an unconstrained linear model with news shocks to the nominal interest rate. The news shocks are used to either peg the expected nominal interest rate or to achieve a targeted change in a long-term yield. For the interest rate peg, the equivalent specification in our setup is to set  $\alpha_j = 1$  for all  $j$  within the forward guidance horizon. In that situation, we would specify news shocks to offset changes in the expected nominal rate due to mean reversion in the discount factor process. When we set  $\alpha_j = 1$  for all  $j$  over long horizons, our solution algorithm does not converge. That result occurs because there is a maximum amount of time that the economy can spend at the ZLB and still deliver a convergent MSV solution [Richter and Throckmorton (2013)].

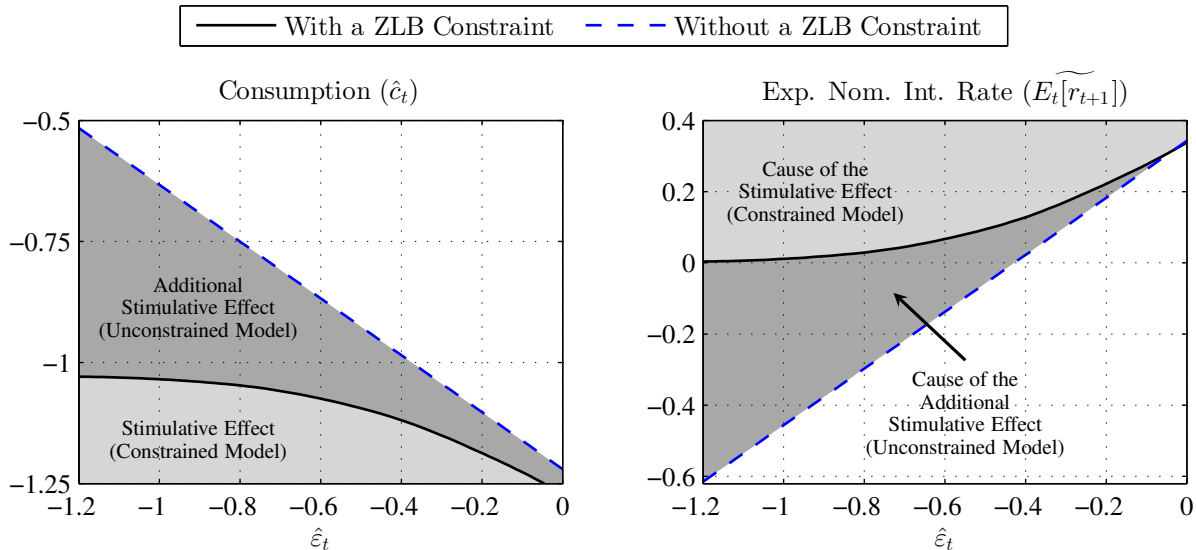


Figure 4: Comparison of decision rules with a ZLB constraint (solid line) and without a ZLB constraint (dashed line) given 1-quarter forward guidance,  $(\alpha_0, \alpha_1) = (0, 1)$ . The decision rules are based on a cross section of the state space where  $\hat{\beta}_t = 1.15$ , which is the minimum value of the discount factor necessary for the ZLB to bind. The horizontal axis displays the news/policy shock, which is specified in percentages. The vertical axis shows the values for consumption, which are in percent deviations from its stochastic steady state and the nominal interest rate, which are in net percentages.

stimulate the economy using 1-quarter forward guidance,  $(\alpha_0, \alpha_1) = (0, 1)$ . Figure 4 compares the responses of key decision rules to that announcement for the model with a ZLB constraint (solid line) and the model without a ZLB constraint (dashed line). The ZLB constraint is critical because it truncates the expected nominal interest rate, which limits the stimulative effect of forward guidance. We utilize 1-quarter forward guidance rather than 1-quarter equal forward guidance so that the stimulative effect is due entirely to changes in the expected nominal rate. Without a ZLB constraint, the expected nominal interest rate can be negative, which leads to an overestimation of the stimulative effect of forward guidance. For example, a  $-50$  basis point news shock in the constrained model generates an expected nominal interest rate of 9 basis points and keeps consumption 1.1 percent below its steady state. The same shock in the unconstrained model, pushes down the expected nominal rate to  $-6$  basis points and raises consumption to 0.92 percent below its steady state. In the unconstrained version of our model, a negative expected nominal interest rate occurs whenever  $\hat{\epsilon}_t < -0.43$ . Therefore, any ZLB analysis that allows the nominal interest rate to fall below zero will significantly overstate the stimulative effect of forward guidance.

The stimulative effect of different types of forward guidance also can be analyzed by examining the impulse responses of key economic variables to those policy shocks. To keep the economy at a stochastic steady state where the ZLB binds, we assume that a series of discount factor shocks holds  $\hat{\beta}_t = 1.15$  so that the nominal interest rate remains at zero. Households, on the other hand, believe the discount factor will gradually revert to its deterministic steady state, so the expected nominal interest rate is above zero. Figure 5 plots the impulse responses to a  $-50$  basis point (2 std. dev.) news shock with no forward guidance (solid line), 1-quarter forward guidance (dashed line), and 1-quarter equal forward guidance (dash-dotted line). In each case, households learn about the monetary policy shock in period 1. With no forward guidance, the shock is unanticipated and

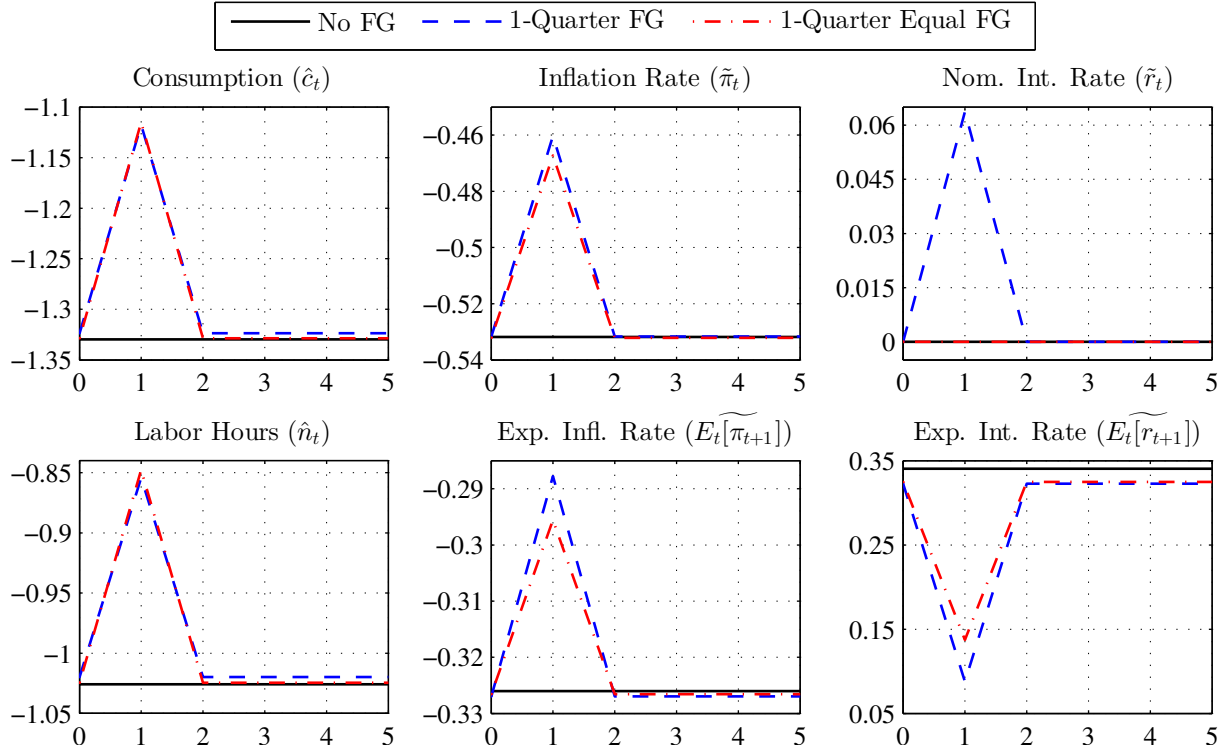


Figure 5: Impulse responses to a  $-50$  basis point monetary policy shock. Three types of monetary policy are examined: No forward guidance,  $(\alpha_0, \alpha_1) = (1, 0)$  (solid line); 1-quarter forward guidance,  $(\alpha_0, \alpha_1) = (0, 1)$  (dashed line); and 1-quarter equal forward guidance,  $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$  (dash-dotted line). Each case is initialized at the stochastic steady state. The discount factor,  $\hat{\beta}_t$ , is set to 1.15 for every period, which is the minimum value necessary for the ZLB to bind. The horizontal axis displays the time period. The vertical axis values of each variable are specified as the percent deviations from its stochastic steady state, except inflation and both interest rates, which are in net percentages.

occurs in period 1. With 1-quarter forward guidance, households are informed about the policy shock, which will hit in period 2. With 1-quarter equal forward guidance, households learn that identical policy/news shocks will impact the economy in periods 1 and 2. In the absence of any policy shock, the high discount factor holds consumption, labor hours, the inflation rate, output, and the current nominal interest rate below their respective steady states.

When there is no forward guidance  $[(\alpha_0, \alpha_1) = (1, 0)]$ , the unanticipated expansionary shock is *not* stimulative because the current nominal interest rate is already at zero and the expected future nominal interest rate remains unchanged. A news shock announced with 1-quarter forward guidance  $[(\alpha_0, \alpha_1) = (0, 1)]$ , in contrast, lowers next period's expected nominal interest rate. That lower rate pushes up current consumption, inflation, output, and labor hours and also raises the current nominal rate due to the feedback effect. Specifically, the period 1 announcement of a  $-50$  basis point monetary policy shock next period raises expected consumption and lowers the expected nominal interest rate for period 2. That change causes households to increase their current consumption and reduce their current labor supply. Firms respond to the increase in demand by raising their prices, output, and labor demand. The additional labor demand dominates the decline in labor supply so that the equilibrium level of labor and the real wage both increase. Our results demonstrate that 1-quarter forward guidance is stimulative, but only if



households expect the economy to recover and exit the ZLB in the near term.

The stimulative effect of 1-quarter forward guidance increases the current nominal interest rate by 6.5 basis points due to the feedback effect. To offset that effect, the central bank could provide an unanticipated expansionary monetary policy shock (via traditional open market operations) in period 1. With 1-quarter equal forward guidance  $[(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ , dash-dotted line], the unanticipated policy shock in period 1 completely counteracts the feedback effect so that the current nominal interest rate remains at zero. The smaller weight on the period 2 news shock, however, reduces the decline in next period's expected nominal interest rate. The effects of the lower current nominal rate and the higher expected future nominal rate essentially offset each other, so that consumption rises by approximately the same amount with 1-quarter equal forward guidance as it does with 1-quarter forward guidance.

**4.2 TWO-QUARTER HORIZON ( $q = 2$ )** This section analyzes the effect of 2-quarter forward guidance at the ZLB. Figure 6 plots the 2-quarter forward guidance decision rules  $[(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$ , dashed line] as a function of the current ( $\hat{\varepsilon}_t$ ) and lagged ( $\hat{\varepsilon}_{t-1}, \hat{\varepsilon}_{t-2}$ ) monetary policy shocks. As a reference, the decision rules for no forward guidance  $[(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$ , solid line] are also displayed. In this paper, 2-quarter forward guidance occurs when households receive news about a future monetary policy shock two periods before that shock impacts the economy.

The stimulative effect of 2-quarter forward guidance resembles the impact of 1-quarter forward guidance in certain situations. For example, the central bank provides 2-quarter forward guidance in period  $t - 2$  about its policy actions in period  $t$ . That policy has nearly the same economic effects in period  $t$  as 1-quarter forward guidance provided in period  $t - 1$ . In both situations, an expansionary policy shock in the current period has no economic effects regardless of when the policy change is announced because the current nominal interest rate is already at zero and the expected nominal rate does not change. A contractionary shock, on the other hand, pushes up the current nominal interest rate, which drives down current consumption, but has no effect on the expected nominal interest rate. A similar comparison can be made between a 2-quarter forward guidance announcement in period  $t - 1$  and a 1-quarter forward guidance announcement in period  $t$ . When the policy is expansionary, households expect a lower nominal interest rate in period  $t + 1$  than they would expect absent forward guidance, which stimulates consumption in period  $t$ .

If households receive information this period that an expansionary monetary policy shock will happen two periods in the future, the response of consumption to that 2-quarter forward guidance will be similar to its response to 1-quarter forward guidance that is announced this period. Given that households prefer a smooth consumption path, the expectation of monetary stimulus in period  $t + 2$  encourages households to raise their consumption in not only period  $t + 2$ , but also in periods  $t$  and  $t + 1$ . The higher demand for consumption in periods  $t$  and  $t + 1$  puts upward pressure on inflation, which pushes up the current nominal interest rate and next period's expected nominal rate via the central bank's policy rule. In other words, the feedback effect impacts both the current nominal interest rate and next period's expected nominal rate.

The 2-quarter forward guidance case is informative because it reveals how the forward guidance horizon affects the economy. One shortcoming of that case is that an expansionary policy shock generates a rise in both the current nominal interest rate and next period's expected rate, which is inconsistent with current monetary policy (i.e.,  $r_t$  and  $E_t[r_{t+1}]$  rise when  $\hat{\varepsilon}_t < 0$ ). In practice, central banks offset increases in the current and expected future rates by promising to keep the nominal interest rate at zero over the entire forward guidance horizon. Our model assumes the

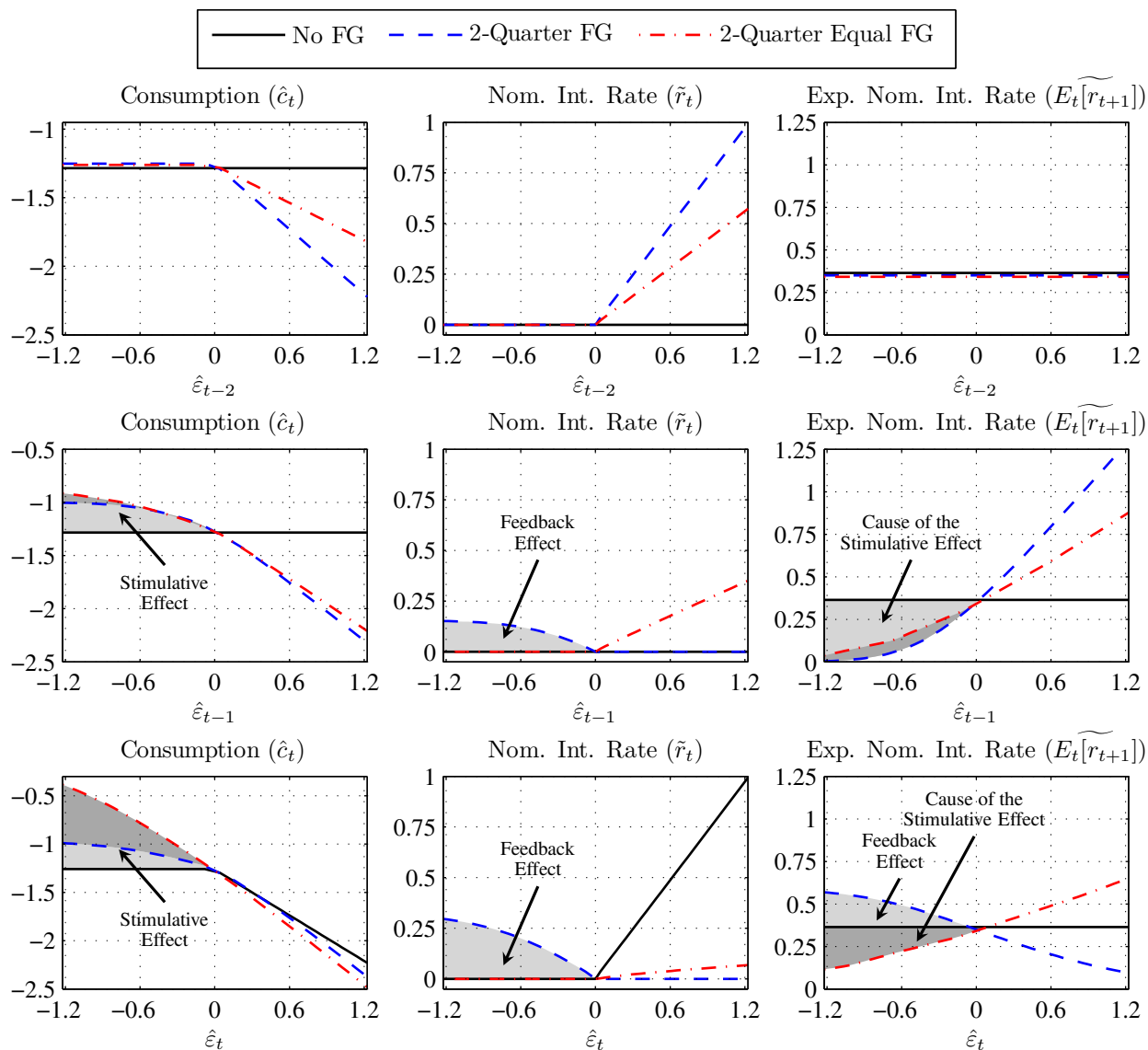


Figure 6: Consumption and the nominal interest rate decision rules as a function of the current and lagged monetary policy shocks with no forward guidance,  $(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$  (solid line); 2-quarter forward guidance,  $(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$  (dashed line); and 2-quarter equal forward guidance,  $(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$  (dash-dotted line). The values are based on a cross section of the state space where  $\hat{\beta}_t = 1.15$ , which is the minimum value necessary for the ZLB to bind. The horizontal axis displays the news/policy shock, which is specified in percentages. The vertical axis shows the values for consumption, which are in percent deviations from its stochastic steady state and the nominal interest rate, which are in net percentages.

central bank announces a plan in period  $t$  to shock the policy rule by an equal amount in periods  $t$ ,  $t + 1$ , and  $t + 2$ . We call that policy 2-quarter equal forward guidance.

Figure 6 also shows the decision rules when households receive 2-quarter equal forward guidance  $[(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})]$ , dash-dotted line]. The specification for the policy shocks keeps the variance of the news process the same for both types of 2-quarter forward guidance. Suppose, for example, households receive forward guidance in period  $t - 1$  about a monetary stimulus over the next two quarters. With 2-quarter forward guidance, there is no period  $t$  policy

shock to offset the feedback effect, so the current nominal interest rate rises. There is a period  $t$  policy shock, however, with 2-quarter equal forward guidance and as a result, the current nominal interest rate remains at zero. That lower nominal interest rate further stimulates current consumption, which allows 2-quarter equal forward guidance to have a larger impact on economic activity.

Substantial differences exist between the two types of 2-quarter forward guidance when the news is communicated in period  $t$ . With 2-quarter equal forward guidance, the central bank announces in period  $t$  that an expansionary monetary policy shock will occur in periods  $t$ ,  $t + 1$ , and  $t + 2$ . The existence of policy shocks in periods  $t$  and  $t + 1$ , which are not present with 2-quarter forward guidance, holds the current nominal interest rate at zero and lowers next period's nominal rate. Thus, the period  $t$  and  $t + 1$  policy shocks eliminate the feedback effects generated by 2-quarter forward guidance. The presence of those policy shocks also more than compensates for the smaller period  $t + 2$  news shock, so that 2-quarter equal forward guidance produces a larger stimulative effect on current consumption than a monetary stimulus with a larger policy shock that only occurs in period  $t + 2$ . For example, a  $-50$  ( $-100$ ) basis point policy shock announced in period  $t$  increases current consumption by 0.24 (0.50) percentage points *more* with 2-quarter equal forward guidance than with 2-quarter forward guidance.

Extending the forward guidance horizon from 1 to 2 quarters does not double the size of its stimulative effect, even if the central bank shocks the policy rate by the same amount in every period (i.e., equal forward guidance). For example, a  $-50$  ( $-100$ ) basis point monetary policy shock during that period increases current consumption by 0.24 (0.37) percent with 1-quarter equal forward guidance and by 0.40 (0.75) percent with 2-quarter equal forward guidance. That finding, however, contradicts much of the literature. The reason our result is different is because of our assumption that the variance of the news process remains constant across alternative forward guidance horizons. If, on the other hand, we follow convention and set  $\{\alpha_j\}_{j=0}^q = 1$ , then the stimulative effect of forward guidance increases as the horizon lengthens. The majority of the additional stimulus is due to the central bank's decision to provide more news and not to the longer horizon.

Many researchers who examine forward guidance do not hold the variance of the news process constant [Laséen and Svensson (2011), Carlstrom et al. (2012), and Del Negro et al. (2012)]. Without that restriction, extending the forward guidance horizon by an additional quarter is the same as increasing the intensity of the news. [Figure 7](#) contrasts key decision rules with 2-quarter equal forward guidance  $[(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})]$ , solid line] to 2-quarter *full* forward guidance  $[(\alpha_0, \alpha_1, \alpha_2) = (1, 1, 1)]$ , dashed line]. Our comparison demonstrates that a higher intensity of news with 2-quarter full forward guidance further stimulates consumption. That finding, however, could erroneously lead one to conclude that the model overstates the stimulative effect of forward guidance. For example, a  $-50$  ( $-100$ ) basis point monetary policy shock with 2-quarter full forward guidance increases consumption from  $-0.86$  ( $-0.50$ ) percent to  $-0.59$  ( $-0.17$ ) percent below its steady state. That result is due to a 7 (8) basis point decline in the 1-quarter ahead expected nominal interest rate and an even larger reduction in the 2-quarter ahead expected rate.

In reality, extending the forward guidance horizon further into the future creates a greater time-inconsistency problem because the central bank has a larger incentive to revert to its optimal policy rule in future periods. If households believe the central bank will renege on its past promises to provide future monetary stimulus, then forward guidance will not have the desired expansionary effects in the current period. The fact that the standard New Keynesian model assumes the central bank is fully credible leads it to overpredict the stimulative effect of forward guidance if households believe the central bank will not follow through on its policy commitments. In contrast, an

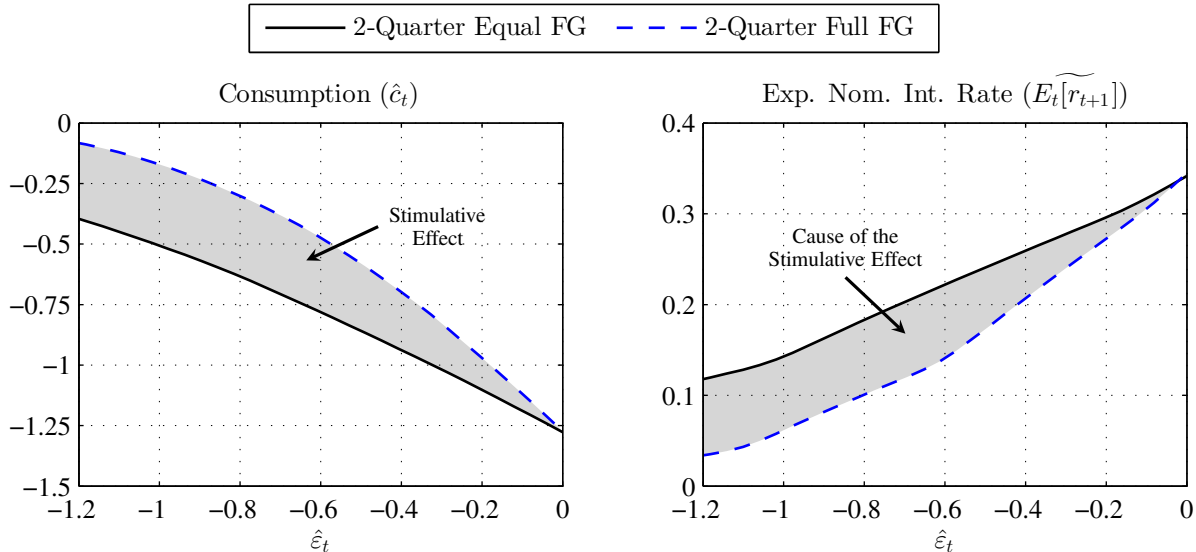


Figure 7: Consumption and the expected nominal interest rate decision rules with 2-quarter equal forward guidance,  $(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$  (solid line), and 2-quarter full forward guidance,  $(\alpha_0, \alpha_1, \alpha_2) = (1, 1, 1)$  (dashed line). The values are based on a cross section of the state space where  $\hat{\beta}_t = 1.15$ , which is the minimum value necessary for the ZLB to bind. The horizontal axis displays the news shock, which is specified in percentages. The vertical axis shows the values for consumption, which are in percent deviations from its stochastic steady state and the nominal interest rate, which are in net percentages.

unexpected decision by the central bank to renege on its previous forward guidance promises will not impact our model’s results until the period household’s learn about the change. That announcement then can be incorporated into our model by assuming that another discretionary news shock hits the economy and reverses the central bank’s previously announced news shocks.

It is important to reiterate that the stimulative effect of forward guidance lessens as the discount factor moves further above its minimum value necessary for the ZLB to bind. As the economy moves deeper into the ZLB region (i.e.,  $\hat{\beta}_t$  moves above and away from 1.15), the probability that the nominal interest rate will rise above zero during the next period declines. The likelihood of leaving the ZLB next period also becomes smaller as households’ expectation of a fast economic recovery fades (i.e.,  $\hat{\beta}_t$  is expected to slowly return to its steady state). In both cases, consumption is expected to be lower in the following period, which dampens any expected increase in next period’s nominal interest rate. Without any meaningful rise in next period’s expected nominal rate, the stimulative effect of forward guidance is extremely limited.

**4.3 LONGER HORIZONS ( $q > 2$ )** This section compares the stimulative effect of forward guidance policies over longer horizons. Our results in sections 4.1 and 4.2 rely on Gauss-Hermite quadrature to evaluate expectations. That approach enables us to obtain an accurate approximation of the decision rules and to quantify the stimulative effect of forward guidance for a *continuous range* of news shocks. The solution algorithm, however, is not numerically feasible at longer forward guidance horizons. Therefore, we reduce the dimensionality of the problem when analyzing longer forward guidance horizons by discretizing the continuous distribution of the news shock process using the method described in Tauchen (1986). Specifically, we examine 3 values for each monetary policy shock,  $(-50, 0, 50)$ , and then calculate the probabilities of each transitional event.

Tauchen’s (1986) method is particularly useful for calculating nonlinear impulse response functions because it enables us to analyze the effects of *specific* shocks to the news process without solving the model across the complete distribution of shocks.<sup>10</sup>

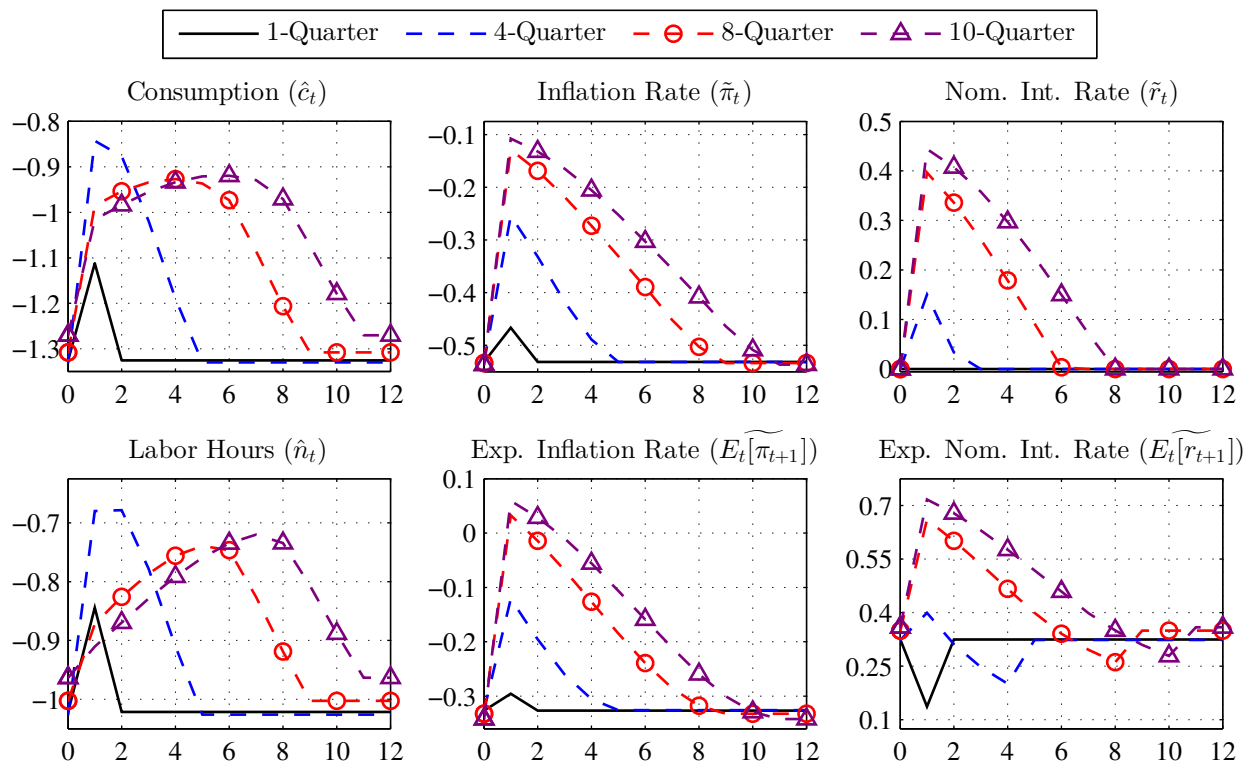


Figure 8: Impulse responses to a  $-50$  basis point monetary policy shock with 1-quarter (solid line), 4-quarter (dashed line), 8-quarter (circle markers), and 10-quarter (triangle markers) forward guidance. All cases are initialized at the stochastic steady state. The discount factor,  $\hat{\beta}_t$ , is set to the minimum value necessary for the ZLB to bind conditional on the forward guidance horizon. The horizontal axis displays the time period. The vertical axis values of each variable are specified as the percent deviations from its stochastic steady state, except inflation and both interest rates, which are in net percentages.

Figure 8 shows the impulse responses to a  $-50$  basis point (2 std. dev.) monetary policy shock that is equally distributed over a 1-quarter (solid line), 4-quarter (dashed line), 8-quarter (circle markers), and 10-quarter (triangle markers) horizon. Specifically, we set  $\alpha_i = \sqrt{1/(q+1)}$  for  $i \in \{0, \dots, q\}$  to keep the variance of the news constant for different values of  $q$ . In all cases, households know about the current and future monetary policy shocks in period 1, but their effects on the economy depend on the forward guidance horizon. In the absence of forward guidance, an expansionary monetary policy shock has no stimulative effect since the nominal interest rate is already at zero. When the central bank provides 1-quarter equal forward guidance, households expect a lower nominal interest rate in period 2, which stimulates the economy in period 1. Over that 1-quarter horizon, the stimulative effect is relatively small and the unexpected policy shock in period 1 is big enough to completely offset the feedback effect on the current nominal interest rate.

Additional periods of forward guidance generate more persistent increases in consumption, labor hours, inflation, and output. That result occurs because households expect a lower future

<sup>10</sup>See Appendix B for further details on how this solution procedure differs from the method used in earlier sections.

nominal interest rate and higher future consumption in every period until the last shock hits the economy in period  $q$ . At longer horizons, the feedback effect from forward guidance dominates the smaller weight on  $\hat{\varepsilon}_t$ , so that the current nominal interest rate increases in period 1. Beyond period  $q$ , forward guidance has no additional effects on the economy. Our findings reveal that equal forward guidance for up to 10 quarters does not cause the stimulative effect to explode or reverse, but rather spreads the effect across the entire horizon.

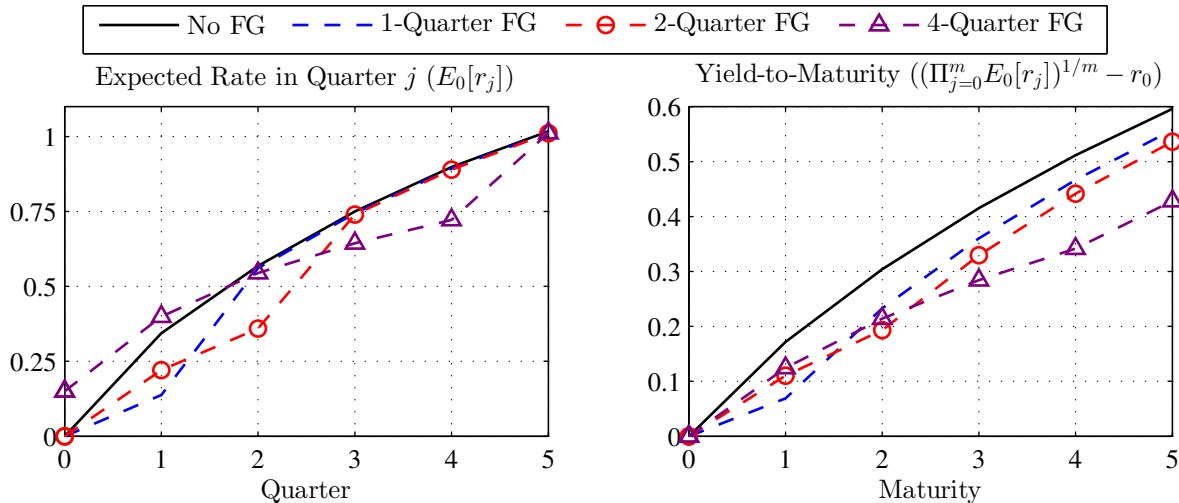


Figure 9: Comparison of expected rates and yields with no forward guidance (solid line) and 1-quarter (dashed line), 2-quarter (circle markers), and 4-quarter (triangle markers) equal forward guidance. The left panel shows the expected increase in the net nominal interest rate  $j$  periods in the future. The right panel shows the yield curves as a percentage point difference from the initial net nominal interest rate. In period 0, households receive news of a  $-50$  basis point monetary policy shock. The moving average coefficients in the news process are set to keep the variance constant across the different horizons where the discount factor,  $\hat{\beta}_t$ , is equal to 1.15, which is the minimum value necessary for the ZLB to bind. The horizontal axis in the left panel displays the quarter while in the right panel it displays the maturity. The vertical axis shows the values for the expected nominal interest rate and the yield curve, which are in net percentages.

Figure 9 compares the paths of the expected future nominal interest rate,  $E_0[r_j]$ , (left panel) and the yield curve (right panel) at various forward guidance horizons.<sup>11</sup> Suppose, for example, the central bank announces in period 0 that it will cut the nominal interest rate by 50 basis points with no forward guidance (solid line), or with 1-quarter (dashed line), 2-quarter (circle markers), or 4-quarter (triangle markers) equal forward guidance. The discount factor,  $\hat{\beta}_t = 1.15$ , is again set to the minimum value necessary for the ZLB to bind. The intensity parameters,  $\alpha_i$ , in the  $q$ -quarter equal forward guidance specification are set to the values needed to keep the variance of the news process constant across the forward guidance horizon (i.e.,  $\alpha_i = \sqrt{1/(q+1)}$  for  $q \in \{0, 1, 2, 4\}$ ).

The left panel of figure 9 shows the impact of forward guidance on the expected nominal interest rate over six quarters. In most cases, forward guidance lowers the expected nominal rate during the forward guidance horizon, but in all cases, it returns to its baseline expected value in the periods beyond that horizon. The baseline expected nominal interest rate is the case with no forward guidance (solid line) because an unanticipated monetary policy shock in period 0 has no economic effects when the current nominal interest rate is already at zero. The expected increase

<sup>11</sup>Swanson and Williams (2013) empirically show how the ZLB affects intermediate- and longer-term yields.



in the nominal interest rate in the no forward guidance specification reflects the expected recovery implied by the belief that the discount factor will revert to its mean. The 1- and 2-quarter equal forward guidance cases lower the expected nominal rates during the forecast horizon but revert to their no forward guidance expected values beginning in periods 2 and 3, respectively. The 4-quarter equal forward guidance horizon behaves just like the 1- and 2-quarter cases, except the policy shock is not strong enough in period 0 to completely eliminate the feedback effect so the current nominal interest rate rises by 15 basis points (see also figure 8).

The right panel of figure 9 illustrates the yield curve for maturities from 0 to 5 quarters in which the yield for each maturity,  $m$ , is calculated as  $(\prod_{j=0}^m E_0[r_j])^{1/m}$ . The stimulative effect of forward guidance is evident as the yield curve flattens with longer horizons.<sup>12</sup> For example, 4-quarter equal forward guidance reduces the expected increase in the 1-year yield (i.e.,  $m = 3$ ) by 13 basis points. Beyond the forward guidance horizon, the yield curves with  $q$ -quarter forward guidance eventually converge to the yield curve with no forward guidance.

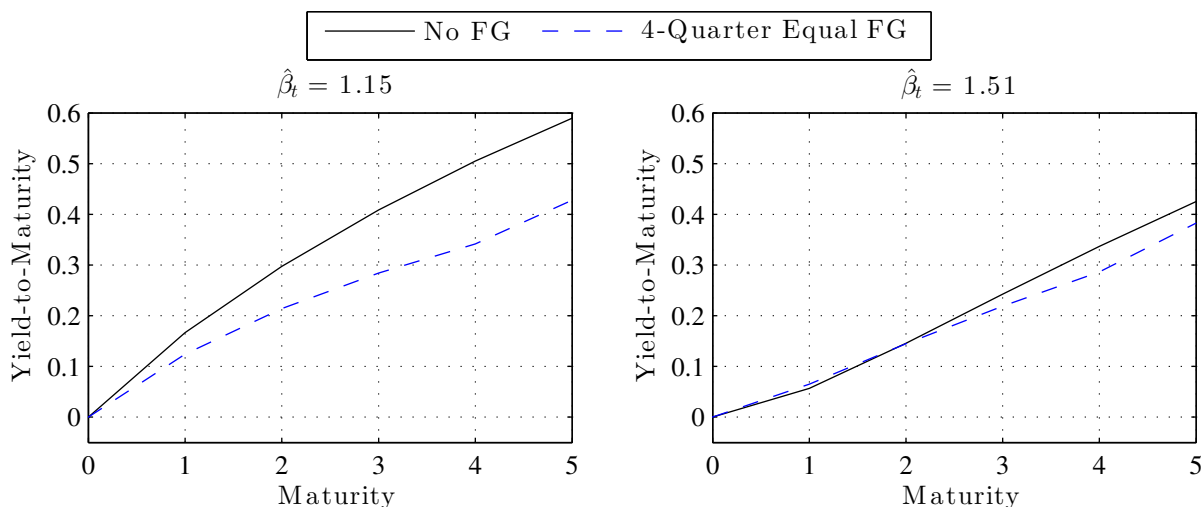


Figure 10: Comparison of yield curves with no forward guidance (solid line) and 4-quarter equal forward guidance (dashed line) at two cross sections of the discount factor:  $\hat{\beta}_t = 1.15$  (left panel) and  $\hat{\beta}_t = 1.51$  (right panel). The yield curves are given as a percentage point difference from the initial net nominal interest rate. In period 0, households receive news of a  $-50$  basis point monetary policy shock. The moving average coefficients in the news process are set to keep the variance constant across the two forward guidance horizons. The horizontal axis displays the maturity. The vertical axis shows the values for the yield curve, which are in net percentages.

Figure 10 compares the effect of forward guidance on the yield curve for two different values of the discount factor. The solid line displays the yield curve when the central bank offers no forward guidance  $[(\alpha_0, \alpha_1) = (1, 0)]$ , while the dashed line shows the yield curve with 4-quarter equal forward guidance  $[\alpha_i = \sqrt{1/5}$  for  $i \in \{0, \dots, 4\}]$ . In the left panel, the discount factor is set to the minimum value necessary for the ZLB to bind ( $\hat{\beta}_t = 1.15$ ). With such a low value, households place a high probability on exiting the ZLB next period. If the central bank provides 4-quarter equal forward guidance, then the expected future nominal interest rates over the forward guidance horizon fall, which flattens the yield curve. In the right panel, the discount factor is set to a high enough value so that households believe the probability of exiting the ZLB in the near

<sup>12</sup>Note that we subtract off the current 1-quarter nominal interest rate,  $r_0$ , from each maturity on the yield curve so that we can observe the degree to which the forward guidance horizon flattens the yield curve.

term is low ( $\hat{\beta}_t = 1.51$ ). With such a high discount factor, an announcement of 4-quarter equal forward guidance has little stimulative effect because households expect that the nominal interest rate will temporarily remain near zero regardless of the news. In that situation, the yield curve with 4-quarter equal forward guidance is similar to the yield curve with no forward guidance.

The results in [figure 10](#) provide a plausible explanation for the Forward Guidance Puzzle. In particular, they show theoretical models can easily overpredict the stimulative effect of forward guidance by assuming households are overly optimistic about future economic conditions. Suppose, for example, the central bank decides to provide a  $-50$  basis point news shock that is equally distributed via 4-quarter equal forward guidance. The response of the yield curve depends on how the nominal interest rate is expected to respond over the next 4 quarters. When households are highly confident that the nominal interest rate will leave its ZLB in the near term [ $\hat{\beta}_t = 1.15$ ], the 4-quarter equal forward guidance shock pushes down the annual yield on the 4-quarter bond by about 50 basis points. That response declines to 10 basis points when the economy is weak enough that households place a high probability on the nominal rate remaining at or near zero [ $\hat{\beta}_t = 1.51$ ].

## 5 A CASE STUDY: FORWARD GUIDANCE IN AUGUST 2011

The FOMC announced in its August 9, 2011 policy statement that it expected the federal funds rate to remain between 0-25 basis points until mid-2013. That statement represented a significant change in policy because it was the first time the FOMC committed to holding its policy rate constant for a fixed period. One factor influencing that decision was the FOMC's expectation that the recovery would be somewhat slower than previously thought just two months prior.<sup>13</sup>

Using Blue Chip survey data from the period, Crump et al. (2013) find that the August 9th FOMC statement changed private forecasts of interest rates, GDP, and inflation. Estimating the effect of that announcement on economic forecasts, however, is complicated by the fact that GDP was revised downward just 11 days before the FOMC's statement was released. To separate the impact of the two events, Crump et al. (2013) compare the consensus forecasts of the federal funds rate from the Blue Chip Financial Forecasts (BCFF) survey to forecasts of the 3-month Treasury bill rate from the Blue Chip Economic Indicators (BCEI) survey.<sup>14</sup>

[Table 1](#) shows the term structure implied by the BCFF and BCEI forecasts. The BCFF forecasts were made on July 20-21 before the GDP revisions were released on July 29th, while the BCEI forecasts were made on August 4-5 just prior to the August 9th FOMC announcement. The difference between the late-July and early-August forecasts is an implicit measure of the impact the GDP revisions had on the economic forecasts. The next BCFF survey forecasts were made on August 24-25. The difference between the BCEI's August 4-5 forecasts and the BCFF's August 24-25 forecasts is an indirect measure of the effect of forward guidance on those forecasts.

Our calculations indicate that the GDP revisions led to a 6 basis point (annualized) decline in the 4-quarter rate and a 13 basis point decline in the 6-quarter rate. The August 9th announcement of forward guidance reduced the 4-quarter and 6-quarter rates by an additional 4 and 18 basis points, respectively. Those results show forward guidance was successful in pushing down the

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<sup>13</sup>Woodford (2012) argues that information about the central bank's policy intentions is more likely to impact market expectations than knowledge about its economic forecasts. The reason is the central bank has undisclosed information about its own policy intentions, whereas it does not enjoy the same advantage when forecasting economic conditions.

<sup>14</sup>The 3-month Treasury bill rate is a good proxy for the federal funds rate, given that the two variables have moved together since the FOMC first announced its federal funds rate target in the early 1990s.

	1Q	2Q	3Q	4Q	5Q	6Q
BCFF (7/20-21)	0.140	0.150	0.187	0.237	0.336	0.459
BCEI 3-mo. T-bill (8/4-5)	0.090	0.110	0.137	0.175	0.240	0.328
BCFF (8/24-25)	0.120	0.125	0.130	0.135	0.138	0.145
Total Change	-0.020	-0.025	-0.057	-0.102	-0.197	-0.313
Change following GDP	-0.050	-0.040	-0.050	-0.062	-0.096	-0.131
Change following FOMC	0.030	0.015	-0.007	-0.040	-0.102	-0.183

Table 1: Term structure implied by Blue Chip consensus forecasts. All values are annualized net interest rates.

yield curve. In addition, the 6-quarter rate was only 2.5 basis points higher than the 1-quarter rate, which indicates households believed the FOMC would keep its promise to leave the policy rate unchanged over the next year and a half.

Table 2 reproduces the 2012 consensus forecasts of GDP and inflation as summarized by Crump et al. (2013). The forecast for 2012 GDP growth dropped 0.3 percentage points after the downward revisions to GDP on July 29th and another 0.3 percentage points following the release of the FOMC’s policy statement on August 9th. The inflation forecast, on the other hand, dropped by a mere 0.1 percentage points following the GDP revisions and remained unchanged after the FOMC’s policy statement release. Therefore, it appears that the economic pessimism expressed in the FOMC’s August 9th policy statement dampened output forecasts more than any stimulative effect from the change in forward guidance. Inflation forecasts were essentially unchanged.

	Pre-GDP	Pre-FOMC	Post-FOMC
2012 real GDP growth rate	2.8	2.5	2.2
2012 CPI growth rate	2.3	2.2	2.2

Table 2: Consensus forecasts reproduced from Crump et al. (2013).

There is evidence to suggest the discount factor was well above one when the FOMC provided forward guidance on August 9th. Wu and Xia (2014), for example, calculate that the “shadow” federal funds rate has been about 1 to 2 percentage points below zero since 2010, which indicates that the discount factor has likely been above one.<sup>15</sup> Our theoretical analysis reveals that the stimulative effect of forward guidance will be limited in this situation, which is consistent with estimates of Crump et al. (2013). Another factor driving the shadow federal funds rate down is the high level of productivity during the period. For example, Fernald (2012) and Gust et al. (2013) find evidence that households have been more patient and more productive since the Great Recession. Gavin et al. (2014) theoretically show that both a high discount factor and a high level of productivity drive the nominal interest rate deep into the ZLB region. That combination of factors then reduces the probability the nominal interest rate will exit the ZLB in the near term and as a result, significantly dampens the stimulative effect of forward guidance.

<sup>15</sup>The “shadow” federal funds rate is the rate that would occur in the absence of the ZLB constraint. Both Krippner (2013) and Bauer and Rudebusch (2014) describe how to estimate the shadow federal funds rate when the ZLB binds. Their estimates also imply that the shadow rate has been well below zero since 2010.

Our model predicts Odyssean forward guidance will reduce longer-term interest rates and push up output given the appropriate economic conditions. An analysis of the data following the August 9th FOMC announcement indicates that longer-term interest rates fell slightly but output declined. To determine why output did not rise, we review the conditions necessary for forward guidance to stimulate the economy: (1) Forward guidance must lower the expected future nominal interest rate; (2) Households must believe the FOMC is committed to its policy; (3) Households cannot interpret the news as a downward revision in the FOMC's economic outlook (i.e., a higher  $\rho_\beta$ ); and (4) Households must place a high probability on exiting the ZLB (i.e., a lower  $\hat{\beta}_t$ ).

An examination of the August 9th FOMC announcement reveals that this case of forward guidance did not have the desired stimulative effect because conditions 3 and 4 were not satisfied. That is, households' expectations about future economic conditions appear to have been dampened by the additional economic pessimism expressed in the policy statement. Empirical studies also indicate that in August 2011 the near-term probability of exiting the ZLB was extremely small. We believe those two characteristics are the primary reasons why output forecasts did not rise as some may have predicted following the August 9th FOMC announcement.

## 6 CONCLUSION

This paper examines the economic effects of Odyssean forward guidance when the nominal interest rate is at the ZLB. In this framework, the central bank conducts forward guidance by promising to keep the future nominal interest rate lower than its policy rule suggests. That policy can stimulate economic activity if households believe future nominal interest rates will rise in the absence of any additional policy shocks. Their expectation is based on the idea that the discount factor is above its steady state and will converge back toward that value. If, on the other hand, households do not expect any meaningful recovery, then the future nominal interest rate will remain at or near zero, which means that forward guidance will have little to no effect on the economy. Therefore, an expectation of a weak recovery or little confidence in the economy dampens the increase in the expected nominal interest rate, which reduces the ability of forward guidance to stimulate current consumption. Those points offer plausible explanations for the Forward Guidance Puzzle.

When a central bank increases its forward guidance horizon, the stimulative effect is spread across a longer period, but its ability to achieve that result depends on credibility. From a policy-maker's perspective, a longer horizon makes it more difficult to assume that people have similar expectations about the economy and more difficult for them to believe that a central bank will continue to provide monetary stimulus that far into the future when the economic fundamentals call for a higher policy rate. If the public does not believe the policy is credible or know the levels of output and inflation at which the short-term interest rate will rise, then the stimulative effect of forward guidance over a longer horizon will be dampened.

The FOMC's recent forward guidance policy likely reduced long-term interest rates [Gürkaynak et al. (2005), Campbell et al. (2012), Moessner (2013), and Swanson and Williams (2013)]. It is unclear, however, how much of that decline was due to the Fed's policy and how much was because of the repeated pessimism about the economy in its policy statements. Our results suggest that if a central bank expresses a more pessimistic economic outlook when communicating Odyssean forward guidance, then its stimulative effect will be much more limited.

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## A NUMERICAL ALGORITHM

A formal description of the numerical algorithm begins by writing the model compactly as

$$\mathbb{E}[f(\mathbf{z}_{t+1}, \mathbf{w}_{t+1}, \mathbf{z}_t, \mathbf{w}_t) | \Omega_t] = 0,$$

where  $f$  is a vector-valued function that contains the equilibrium system,  $\mathbf{z}$  is a vector of exogenous variables,  $\mathbf{w}$  is a vector of endogenous variables, and  $\Omega_t = \{M, P, \mathbf{z}_t\}$  is households' information set in period  $t$ , which contains the structural model,  $M$ , its parameters,  $P$ , and the state vector,  $\mathbf{z}$ . With 1-quarter forward guidance,  $\mathbf{z}_t = (\varepsilon_{t-1}, \varepsilon_t, \beta_t)$  and with 2-quarter forward guidance,  $\mathbf{z}_t = (\varepsilon_{t-2}, \varepsilon_{t-1}, \varepsilon_t, \beta_t)$ . Independent of the forward guidance horizon,  $\mathbf{w} = (c, \pi, y, n, w, r)$ .

Policy function iteration approximates the vector of decision rules,  $\Phi$ , as a function of the state vector,  $\mathbf{z}$ . The time-invariant decision rules for the exogenous model are

$$\underbrace{\Phi(\mathbf{z}_t)}_{\text{True RE Solution}} \approx \underbrace{\hat{\Phi}(\mathbf{z}_t)}_{\text{Approximating Function}}.$$

We choose to iterate on  $\Phi = (c, \pi)$  so that we can easily solve for future variables that enter the households' expectations using  $f$ . Each state variable in  $\mathbf{z}$  is discretized into  $N^d$  points, where  $d \in \{1, \dots, D\}$  and  $D$  is the dimension of the state space. Thus, the discretized state space contains  $N = \prod_{d=1}^D N^d$  nodes. We set the bounds of stochastic state variables to encompass 99.999 percent of the probability mass of the distribution. We specify 61 grid points for each continuous state variable and use 15 Gauss-Hermite nodes for each continuous shock (with 1-quarter forward guidance  $N = 226,981$  and with 2-quarter forward guidance  $N = 13,845,841$ ). Those techniques minimize extrapolation and ensure that the location of the kink in the decision rules is accurate.

The following outline summarizes the policy function algorithm we employ. Let  $i \in \{0, \dots, I\}$  index the iterations of the algorithm and  $n \in \{1, \dots, N\}$  index the nodes.

1. Obtain initial conjectures for the approximating functions,  $\hat{c}_0$  and  $\hat{\pi}_0$ , on each node, from the log-linear model without the ZLB imposed. We use `gensys.m` to obtain those conjectures.
2. For  $i \in \{1, \dots, I\}$ , implement the following steps:
  - (a) On each node, solve for  $\{r_t, w_t\}$  given  $\hat{c}_{i-1}(\mathbf{z}_t^n)$  and  $\hat{\pi}_{i-1}(\mathbf{z}_t^n)$  with the ZLB imposed.
  - (b) Linearly interpolate  $\{c_{t+1}, \pi_{t+1}\}$  given  $\{\varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$  (1-quarter forward guidance) and  $\{\varepsilon_{t-1}, \varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$  (2-quarter forward guidance). Each of the  $M$  pairs of  $\{\varepsilon_{t+1}^m, \beta_{t+1}^m\}$  are Gauss-Hermite quadrature nodes. We use Gauss-Hermite quadrature to numerically integrate, since it is very accurate for normally distributed shocks. We use piecewise linear interpolation to approximate future variables that show up in expectation, since this approach more accurately captures the kink in the decision rules than continuous functions such as cubic splines or Chebyshev polynomials.<sup>16</sup>

<sup>16</sup>Aruoba and Schorfheide (2013) use a linear combination of two Chebyshev polynomials—one that captures the dynamics when the ZLB binds and one that captures the dynamics when the Taylor principle holds. Although that approach is more accurate than using one Chebyshev polynomial, there is no guarantee that it will accurately locate the kink. Moreover, Richter et al. (2013) show that Chebyshev polynomials can lead to large approximation errors due to extrapolation. With linear interpolation, a dense state space leads to more predictable extrapolation and more accurately locates the kink.

- (c) On each node, solve for the remaining  $t + 1$  variables,  $\{y_{t+1}^m, c_{t+1}^m\}_{m=1}^M$ , which enter the expectation operators.
- (d) We use the nonlinear solver, `csolve.m`, to minimize the Euler equation errors. On each node, numerically integrate to approximate the expectation operators,

$$\mathbb{E} [f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n) | \Omega_t] \approx \frac{1}{\sqrt{\pi}} \sum_{m=1}^M f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n) \phi(\varepsilon_{t+1}^m, \beta_{t+1}^m),$$

where  $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$ , and  $\phi$  are the respective Gauss-Hermite weights. The superscripts on  $\mathbf{x}$  indicate which realizations of the state variables are used to compute expectations. The nonlinear solver searches for  $\hat{c}_i(\mathbf{z}_t^n)$  and  $\hat{\pi}_i(\mathbf{z}_t^n)$  so that the Euler equation errors are less than  $1^{-4}$  on each node.

3. Define  $\text{maxdist}_i \equiv \max\{|\hat{c}_i - \hat{c}_{i-1}|, |\hat{\pi}_i - \hat{\pi}_{i-1}|\}$ . Repeat the steps in [item 2](#) until  $\text{maxdist}_i < 1^{-13}$  for all  $n$  and for 10 consecutive iterations.

To provide evidence that the solution is unique, we randomly perturb the converged decision rules and check that the algorithm converges back to the same solution.

## B COMPUTING LONGER HORIZONS

To solve the model with longer forward guidance horizons, we simplify the numerical problem by discretizing the lags in the monetary policy news process with only 3 points each. That simplification is suitable for examining the effects of a *specific shock* with nonlinear impulse response functions, but would not be suitable for showing cross sections of the entire solution across the state space, which demonstrate the effects of *many* degrees of news. We employ the method outlined in Tauchen (1986) to obtain those points and the corresponding weights for numerical integration. Independent of the forward guidance horizon, the state vector is  $\mathbf{z} = (\beta, s_0, s_1)$ .  $s_0 \in \{1, 2, 3\}$  determines the realization of the monetary policy shock,  $\varepsilon_t$ , such that

$$\varepsilon_t = \begin{cases} -0.005 & \text{for } s_{0,t} = 1 \\ 0 & \text{for } s_{0,t} = 2, \\ 0.005 & \text{for } s_{0,t} = 3 \end{cases},$$

which is consistent with the  $-50$  basis point (2 std. dev.) news shock that we consider in the impulse response functions throughout the paper.  $s_1 \in \{1, 2, \dots, 3^q - 1, 3^q\}$  determines the realization of the lagged shocks in the news process, where  $q$  is the length of the forward guidance horizon. For example, if  $q = 1$ , then  $\varepsilon_{t-1} \in \{-0.005, 0, 0.005\}$ , and if  $q = 2$ , then

$$(\varepsilon_{t-1}, \varepsilon_{t-2}) \in \{(-0.005, -0.005), (0, -0.005), (0.005, -0.005), \dots, (0, 0.005), (0.005, 0.005)\}.$$

The transition matrix for  $s_{0,t}$  is ergodic and is characterized by a single vector of probabilities,

$$P = (\lambda_1, \lambda_2, \lambda_3) = (0.1587, 0.6827, 0.1587),$$

where  $\lambda_k = \Pr(s_{0,t+1} = k)$ .

We discretize the state variable  $\beta$  with 151 points so that the total number of nodes in the state space is  $N = 151 \times 3 \times 3^q$ . For example, if the forward guidance horizon is 10 quarters, then there are 26,749,197 nodes. Expectations formation is given by

$$\mathbb{E} [f(\mathbf{x}_{t+1}^k, \mathbf{x}_t^n) | \Omega_t] \approx \sum_{k=1}^3 \lambda_k \frac{1}{\pi} \sum_{m=1}^M f(\mathbf{x}_{t+1}^k, \mathbf{x}_t^n) \phi(\beta_{t+1}^m),$$

where  $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$ , and  $\phi$  are the Gauss-Hermite weights to numerically integrate across realizations of  $\beta_{t+1}$ . The superscript  $k$  on  $\mathbf{x}$  indicates the realization of  $s_{0,t+1}$ . The policy function iteration algorithm is otherwise the same as outlined in [Appendix A](#).