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THE DISTRIBUTION OF CONTRACT DURATIONS ACROSS FIRMS

A UNIFIED FRAMEWORK FOR UNDERSTANDING AND COMPARING DYNAMIC WAGE AND PRICE SETTING MODELS

by Huw Dixon

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Abstract

This paper shows how any steady state distribution of ages and related hazard rates can be represented as a distribution across firms of completed contract lengths. The distribution is consistent with a Generalised Taylor Economy or a Generalised Calvo model with duration dependent reset probabilities. Equivalent distributions have different degrees of forward lookingness and imply different behaviour in response to monetary shocks. We also interpret data on the proportions of firms changing price in a period, and the resultant range of average contract lengths.

JEL: E50.

Keywords: Contract length, steady state, hazard rate, Calvo, Taylor.

Non-Technical Summary.

The concept of nominal rigidity captures the frequency with which wage or price-setters (hereafter "firms") review and reset wages and prices. In a perfect market, wages and prices will be perfectly flexible and vary so as to be at their optimal level reflecting the current demand and cost conditions facing the firm. The current generation of models used in analyzing the effects of monetary policy (the new neoclassical synthesis models) adopt a model of inertia based on a variety of dynamic pricing models. The most commonly used pricing models are the Taylor model of staggered contracts of a fixed and known duration and the Calvo model where the contract length is random (both assume that the wage or price is fixed over the contract length).

The purpose of this paper is to provide a unified framework for modelling the distribution of contract durations which will be comparable across different types of dynamic pricing models and which will give a guide as to how to interpret empirical data.

If we look at an economy, we will observe some wages and prices changing in each period and a resultant distribution of durations. In steady state this distribution is the same in each period. There are four different ways of looking at the same steady state distribution of contract durations. First, we can take a census at a point in time and look at how long the wages and prices at that moment have been in force: that is, we measure the age distribution of contracts. Second, we can look at the hazard rates for various ages, the proportion of contracts which terminate at each age. Third, we can look at the population of contracts and their distribution. Lastly, we introduce the new concept of *the distribution of contracts durations across firms DAF*. This takes the same cross-section at a moment in time as the age distribution, but looks at the distribution of completed contract lengths associated with it, rather than the age. The average age of contracts in cross section will be about half the completed lifetime.

This paper argues that the DAF is the relevant way of understanding nominal rigidity: because it is firms that set prices or wages, we need to measure nominal rigidity across firms.

A commonly used framework for measuring nominal rigidity is to look at the distribution of contract durations. A simple example will show the *DAF* differs from this. Suppose we have two firms: F1 sets a different price each period, F4 once every four periods. Suppose we take a four period sample. We will observe 5 contracts or price-spells: four for firm F1 and one for F4[1]. If we take the average contract length across contracts we get 8/5 priods. However, if we average across firms we get the much longer 3 periods. I believe that many existing studies use the statistical methodology developed for the distribution of contracts not for the *DAF*, and hence have not captured the correct measure of nominal rigidity.

In this paper, we develop a series of identities by which from any of the four ways of representing the steady state you can recover the other three **(**Proposition 1 and Corollary 1).

We then link the statistical models to theoretical pricing models. Firstly, we generalise the standard Taylor model to allow for a distribution of known contract lengths, the Generalised Taylor Economy (*GTE*): this corresponds to the *DAF*. Secondly, the Generalised Calvo model with duration dependent reset probabilities corresponds to the Hazard rate representation. We are then in a position to compare different pricing models for the same distribution of contract lengths. We use the example of the Multiple Calvo model (one where there are many sectors, each with a sector specific Calvo reset probability), showing how the same distribution can be modelled as a GTE (where firms know the contract length ex ante) or as a Generalised Calvo model (where the contract lengths are uncertain). We find that for a given distribution of contract lengths, the GTE is more myopic than the GC or MC which put the same weight on the future.

[1] We assume that the contract commences in the first period.

The methodology is applied to the Bils-Klenow (2004) data set wuich gives the average proportion of firms resetting prices for 354 categories used in the US CPI for the period 1995-7. We find that the mean contract length across firms is 4.4 quarters, which fits in well with the notion that the average duration of a price is 4 quarters and is the correct measure of nominal rigidity. If on the other hand you look at the mean contract length across contracts, you get only 2.7 quarters: this is far too short and means that there is over-sampling of short contracts. We also ask what is the shortest mean contract length consistent with a given proportion of firms changing prices: we find that this is the reciprocal of the proportion (proposition 3) and that the maximum mean is proportional to the longest contract length (Proposition 4). Lastly, we generate the Impulse-response functions (*IR*) for a simple model and compare them for the B-K distribution: we find the *IR* of the *MC* and the *GC* are similar; that of the *GTE* reflects is more myopic pricing rule.

The majority of theoretical calibrations and existing empirical work on nominal rigidity are using the wrong statistical framework for measuring nominal rigidity. The identities put forward in this paper provide a fuller understanding of how pricing models and data can be linked together.

1 Introduction

Dynamic pricing and wage-setting models have become central to macroeconomic modelling in the new neoclassical synthesis approach: they are the way that *nominal rigidity* is introduced into the macroeconomic system[1]. The concept of nominal rigidity captures in some sense the frequency with which wage and/or price setters reset or review wages and prices. It has become apparent that different models of pricing have different implications for matters such as the persistence of output, prices and inflation to monetary shocks. The question arises as to how the implications of models are linked to the frequency with which wages or prices are reset and the resultant distribution of contract lengths (where contract length refers to a completed wage or price spell). In this paper I develop a unified approach which can be used to understand and compare the distribution of contract durations implied by models of price and wage-setting, and also empirical data on pricing. The key point is that since we seek to understand nominal rigidity, we want to measure the degree of price stickiness across wage and price setters ("firms" hereafter): we want to know the average frequency of price adjustment and mean duration of price or wage spells at the "firm" level. Much of existing work on nominal rigidity has adopted a statistical framework that does not take into account that prices are set by firms, but treats each price spell as an entity in itself: this gives rise to a mis-measurement of nominal rigidity and has resulted in failing to compare different pricing models consistently (see for example Dixon and Kara 2006a).

We start from the idea of modelling the class of all steady state distributions of durations across a given population (in this case, the firms or unions that set prices or wages): we call this the distribution of durations across firms (DAF). In steady state there are three equivalent ways of interpreting the distribution of durations. First there is the cross-sectional distribution of *ages*: how long has the price or wage contract lasted until now? This is like the population census. Note that since there is one price set by each firm, by taking a cross-section we are simultaneously finding the distribution of ages across current price spells and across firms. Second, we can look at the distribution in terms of *survival probabilities*: from the cross-section of ages, what is the probability of progressing from one age to the next one. A

[1] See Goodfriend and King (1997), Erceg (1997), Clarida, Gertler and Gali (1999), Ascari (2000) and Huang and Liu (2002) inter alia..

third perspective is to take a cross-section of contracts that start at a point in time, and find the distribution of completed contract lengths to which this gives rise. In effect, this last distribution is not taken across all firms, but only across those firms which reset wage or price: it is essentially a cohort distribution. Lastly, we can look at the cross-section of contracts in steady-state across firms and ask what is the distribution of completed contract lengths (lifetimes) of this cross-section: in effect we are associating each firm with a (completed) contract length to get a distribution of completed contract lengths across firms. The first three concepts (distribution of ages, hazard rates and distribution across new starters) are very well understood in statistics, being basic tools in demography, evolutionary biology and elsewhere. The last concept, the distribution of completed durations across the population of firms is a more novel concept, but it is the key to understanding dynamic wage and price setting models since it is this concept that determines nominal rigidity. The contribution of this paper is to provide a set of steady state indentities between these four different ways of looking at the underlying steady state distribution: we can start from any of the first three concepts to arrive at the fourth which corresponds to nominal rigidity.

In order to understand the difference between the DAF and the distribution across new contracts, we can consider the following simple example. Four firms reset price every month. Another four firms reset price once a year: one firm in each quarter. Suppose we take the cohort of firms that reset price in January. There are 5 firms resetting prices: the average length of contracts from this cohort is 8/3 months. If we look at the distribution of contract lengths across all firms, the average duration is 7 months. Clearly, if we want to obtain a sensible measure of nominal rigidity, what matters is the behaviour of all firms which indicates that we should use the DAF, not the distribution across new contracts. However, in calibrating the Calvo model, existing studies tend to use the mean of the distribution across new contracts, not across all firms: hence the notion that a reset probability of 0.25 corresponds to an average contract length of 4 quarters - the average across new contracts is indeed 4 quarters, but across all firms is 7 quarters.

We link the statistical analysis of contracts across firms to general models of price and wage setting. First, the Generalized Taylor Economy (GTE) introduced in Kara and Dixon (2005), which starts from the distribution of completed contract lengths (lifetimes). There are many sectors, each with sector specific contract lengths. The simple Taylor economy where all contract lengths are the same is a special case of the GTE. Secondly, adopting the hazard rate perspective, we take the Calvo approach, where contract lengths are stochastic with a reset probability which may be constant (as in the classical Calvo model) or duration dependent (Wolman 1999). We show that the Calvo model with duration dependent reset probabilities (denoted as the *Generalized Calvo model* GC) is coextensive with the set of all steady state distributions: each possible steady state age distribution has exactly one GC and one GTE which corresponds to it. Hence, using this framework, we are able to compare the different models of pricing *for any given distribution of durations across firms*. This enables us to isolate the precise effect of the pricing model as opposed to the difference in the distribution of contract lengths. As Dixon and Kara (2006a) showed, existing comparisons of simple Taylor and simple Calvo models of pricing have failed to even ensure that the mean contract lengths are the same, let alone the overall distribution of contract lengths across firms (see for example Kiley 2002).

It is widely recognized that there is a variety of pricing or wage-setting behaviour in most economies. This raises the question of aggregation: if we seek to represent the economy with a particular model, is the model itself consistent with this heterogeneity? This paper shows that both the GTE and GC are closed under aggregation: if we combine two economies represented by a GTE, the resultant economy will also be a GTE. Likewise the GC. More importantly, we show that this is not the case for either the simple Taylor or Calvo models. If there is heterogeneity in the economy, then it cannot consistently be represented as a simple Taylor or Calvo process (except possibly as a dubious approximation). However, another generalization of the Calvo idea, the Multiple Calvo economy MC is closed under aggregation. In the MC economy, there are many sectors, each with a sector specific reset probability.

Given that we have a particular distribution of durations, what difference does the pricing model make? Following the analysis of Dixon and Kara (2005), the concept of Forward Lookingness (FL) is employed: how far on average do agents look forward (what is the weighted mean number of periods price setters look forward when the set their price?). We find that in the GTE model, firms on average are more myopic than in the GC model for a give distribution of durations. This leads to observable differences in impulse response functions in response to monetary shocks.

We also apply this approach to the Bils-Klenow data set (Bils and Klenow 2004). From the sectoral data for the proportion of changes in prices per month we are able to construct the average length of contracts under the hypothesis that there is a calvo process in each sector, and also find that the shortest possible mean duration is achieved by the assumption that there is the simplest GTE consistent with the observed proportion, which consists of one or two consecutive contract durations that yield the observed proportion of prices changing (Proposition 3). The longest possible mean is proportional to the longest possible contract length (Proposition 4). This paper provides not only a simple and transparent discrete time framework for understanding nominal price rigidity in dynamic macromodels, but also indicates how empirical evidence from price data can be applied in a consistent and relevant manner.

In section 2 we review the well known facts about the steady state distribution of ages and hazard rates and durations across new contracts. We then introduce the new concept of the distribution of durations across firms and show how all four concepts are related by simple identities which are spreadsheet friendly. In section 3, we link the concepts to different models of pricing. In section 4 the issue of aggregation is considered. In section 5 we analyze the different pricing models in terms of forward lookingness and compare the mean reset prices. In section 6, we implement these ideas using the Bils-Klenow data set.

2 Steady State Distributions of Durations across Firms.

We will consider the steady-state demographics of contracts in terms of their durations. The *lifetime* of a contract is how long it lasts from its start to its finish, a *completed* duration. The *age* of a contract at time t is how long it has been in force since it started. The age is a duration which may or may not be completed. We will first review the well known representation of steady state durations by the related concepts of the *age distribution* and *hazard rates* (see for example, Kiefer 1988).

There is a continuum agents (we will call them firms here) f which set wages or prices represented by the unit interval $f \in [0, 1]$. In steady state we can take a cross-section at time t and measure the age distribution[2]: α_j^s is the proportion of firms which have contracts age j, $\left\{\alpha_j^s\right\}_{j=1}^F$ where F is

[2] In Demography, this is given the acronym SAD.

the oldest age in steady state[3]. In steady state, the distribution of ages is monotonic: you cannot have more older people than younger, since to become old you must first be young. Hence the set of all possible steady state age distributions is given by the following subset of the $(F-1)$ unit simplex Δ^{F-1} :

$$\Delta_M^{F-1} = \left\{ \boldsymbol{\alpha}^s \in \Delta^{F-1} : \alpha_j^s \geq \alpha_{j+1}^s \right\}$$

An alternative way of looking at the steady state distribution of durations is in terms of the *hazard rate*. The hazard rate at a particular age is the proportion of contracts at age i which do not last any longer (contracts which end at age i, people who die at age i). Hence the hazard rate can be defined in terms of the age distribution: given the distribution of ages in steady-state $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$, the corresponding vector of hazard rates[4] $\boldsymbol{\omega} \in [0,1)^{F-1}$ is given by:

$$\omega_i = \frac{\alpha_i^s - \alpha_{i+1}^s}{\alpha_i^s}; i = 1...(F-1) \tag{1}$$

Corresponding to the idea of a hazard rate is that of the *survival probability,* the probability at birth that the price survives for at least i periods, with $\Omega_1 = 1$ and for $i > 1$

$$\Omega_i = \Pi_{\kappa=1}^{i-1}(1-\omega_\kappa)$$

and we define the sum of survival probabilities Σ_Ω and its reciprocal $\bar{\omega}$:

$$\Sigma_\Omega = \sum_{i=1}^F \Omega_i \quad \bar{\omega} = \Sigma_\Omega^{-1}$$

Clearly, we can invert (1): we have $F-1$ equations. Hence:

Observation 1 given $\boldsymbol{\omega} \in [0,1)^{F-1}$, there exists a unique corresponding age profile $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$ given by:

$$\alpha_i^s = \bar{\omega}\Omega_i \quad i = 1...F.$$

[3]In some theoretical applications such as the Calvo model of pricing, there may be infinite lifetimes. The analysis presented is consistent with that, although for all practical applications a finite maximum is required.

[4]Since the maximum length is F, without loss of genersality we set $\omega_F = 1$. If $\omega_i = 1$ for some $i < F$, then i is the maximum duration and subsequent hazard rates become irrelevant. This leads to trivial non-uniqueness. We therefore define F as the *shortest* duration with a reset probability of 1. Hence for $i < F$, $\omega_i \in [0,1)$.

Given the flow of new contracts, the proportion surviving to age i is $\Omega_i : \bar{\omega} = \Sigma_\Omega^{-1}$ ensures adding up. From observation 1 $\alpha_1^s = \bar{\omega}$. From the definition of hazard rates and Observation 1 we can move from an age distribution $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$ to the hazard profile and vice versa.[5]

2.1 The Distribution of Completed Durations across Firms.

Given a steady-state age distribution $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$, we can ask what is the corresponding distribution of *completed* durations or lifetimes across firms $\boldsymbol{\alpha} \in \Delta^{F-1}$. Note, we are asking for the distribution *across firms (DAF).* There is a unit interval of firms: each firm sets one price. When we measure the population shares α_i, we are measuring across firms, just as we do when we take the age distribution. We are seeking to answer the question "what is the distribution of completed across the population of firms". The population of firms does not vary over time[6], and that whilst some firms change price frequently and some infrequently, each individual firm over time has an average contract length, and the average in the economy is the average over the stock of firms. *It is this that corresponds to the concept of price-stickiness*[7].

We now derive the main results of the paper, which provides a framework of steady state identities which links the three familiar concepts of age distribution, hazard rate and distribution across contracts to the distribution of completed contract lengths across firms, and hence to nominal rigidity of prices and wages. First, we consider the relationship between the Hazard profile and the distribution of contract durations across firms.

Proposition 1. (a) Consider any Hazard profile $\boldsymbol{\omega} \in [0,1)^{F-1}$. There exists a unique distribution of lifetimes across firms corresponding to $\boldsymbol{\omega}$, $\boldsymbol{\alpha} \in \Delta^{F-1}$ where:

$$\alpha_i = \bar{\omega}.i.\omega_i.\Omega_i \text{: } i = 1...F \tag{2}$$

[5] This relationship is one of the building blocks of Life Tables (Chiang 1984), which are put to a variety of uses by demographers, actuaries and biologists.

[6] This comes from the assumption of a steady state.

[7] The *DAF* is similar to the concept of the distribution of contract lengths across workers employed by Taylor (1993, page 36) in his analysis of non-synchronised wage-setting with different contract lengths.

(b) Consider any distribution of contract lengths across firms given by $\boldsymbol{\alpha} \in \Delta^{F-1}$. There exists a unique hazard profile that will generate this distribution in steady state $\boldsymbol{\omega} \in [0,1)^{F-1}$ where:

$$\omega_i = \frac{\alpha_i}{i}\left(\sum_{j=i}^{F}\frac{a_j}{j}\right)^{-1}$$

All proofs are in the appendix. Now, since there is a 1-1 relationship between the hazard profile and the distribution of contract lengths across firms, and in addition we know that there is a 1-1 relation between the hazard profile and the age distribution, it follows that there must be a 1-1 relationship between the age distribution and the distribution of completed contract lengths[8].

Corollary 1 (a) Consider a steady-state age distribution $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$. There exists a unique distribution of lifetimes across firms $\boldsymbol{\alpha} \in \Delta^{F-1}$ which corresponds to $\boldsymbol{\alpha}^s$, where

$$\begin{aligned} \alpha_i &= i\left(\alpha_i^s - \alpha_{i+1}^s\right) \quad i = 1..F-1 \\ \alpha_F &= F\alpha_F^s \end{aligned} \tag{3}$$

(b) Given a distribution of steady-state completed lifetimes across firms, $\boldsymbol{\alpha} \in \Delta^{F-1}$, there exists a unique $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$ corresponding to $\boldsymbol{\alpha}$

$$\alpha_j^s = \sum_{i=j}^{F}\frac{\alpha_i}{i} \qquad j = 1...F \tag{4}$$

Lastly, we can also ask for a given distribution of contracts across firms $\boldsymbol{\alpha} \in \Delta^{F-1}$, or age distribution $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$or hazard profile $\boldsymbol{\omega} \in [0,1)^{F-1}$, what is the corresponding distribution of durations taken across the total population of *contracts* $\boldsymbol{\alpha}^d \in \Delta^{F-1}$. These are:

$$\alpha_i^d = \frac{\alpha_i}{i.\bar{\omega}} \tag{5a}$$

$$= \frac{\left(\alpha_i^s - \alpha_{i+1}^s\right)}{\bar{\omega}} \tag{5b}$$

$$= \omega_i.\Omega_i \tag{5c}$$

[8]Note that Corollary 1 is essentially a generalisation of Taylor's 4 quarter example (1993, p37). Taylor's notation for the distribution of ages is π_{i-1} rather than α_i^s here: both notations have α_i for the distribution of durations across workers/firms.

The distribution of durations across contracts is the same as the distribution across firms resetting prices. The more frequent price setters (shorter contracts) have a higher representation relative to longer contracts. Note that the rhs denominator of (5a) is the product of the contract length and the proportion of firms resetting price. For the values of $i < \bar{\omega}^{-1}$, the share of the duration i is greater across contracts than firms: for larger $i > \bar{\omega}^{-1}$ the share across contracts is less than the share across firms. Using equation ($5a$), we can move simply between the distributions across contracts and across firms. From ($5a$) , the mean length across all contracts $\bar{d}$ (which also equals the life expectancy at birth) is:

$$\bar{d} = \sum_{i=1}^{F} i.\alpha_i^d = \bar{\omega}^{-1} \tag{6}$$

The mean contract length taken over all contracts in steady state is the reciprocal of the proportion of firms resetting price. This is precisely the "frequency based" estimate of mean contract length that has been commonly used empirically (see for example Bils and Klenow 2004, Barhad and Eden 2004) and also for calibration purposes in models of price-stickiness (for example, Clarida et al 1999 p.1666). However, it should be clear that this is the mean of the wrong distribution: it is the mean across firms resetting price, not all firms. *There is clear length biased sampling: the shorter contracts (more frequent price resetters) are oversampled, resulting in an underestimate of nominal rigidity.*

In the study of unemployment, each spell of unemployment is treated as an observation and the identity of the person involved is irrelevant. Hence the focus in the unemployment literature on the duration of unemployment has been on the flow of new spells of unemployment and how long they will last, rather than on the stock of unemployed[9]. In demography and evolutionary biology, each duration corresponds to a single individual, hence since people only live once, the distribution across people is exactly the same as the distribution across individual durations: hence the focus here is also on cohort studies, exploring the distribution of ages, hazard rates and lifetimes across people born at the same time. The key difference in this paper arises because we are interested in the pricing behaviour of *firms*: hence whilst the

[9]However, Akerlof and Main (1981) did suggest using the average duration across all the unemployed as an important indicator (see the ensuing debate Carlson and Horrigan 1983, Akerlof and Main 1983).

flow of new contracts is of interest, the average duration of contracts across the stock of firms is what determines price stickiness.

2.2 Evidence from micro data: prices are stickier than we thought!

There are now several studies using micro price data: in particular the *Inflation Persistence Network* (IPN) across the Eurozone has been particularly comprehensive[10]. These studies adopt a common methodology using monthly micro CPI data across several countries. Here we will consider Alvarez and Hernando (2004) for Spain (covering 1994-2003), Veronese et al (2005) for Italy (covering1996-2003), Baudry et al (2004) for France (covering 1994-2003). All these studies have data on individual products sold at individual outlets. Their terminology of a "price spell" (analogous to unemployment spell) is the same as our lifetime of a contract. They also have *trajectories* for prices: this is the sequence of price spells for a product at an individual outlet. We can think of each trajectory as analogous to the sequence of price contracts for an individual firm. These papers all provide estimates of the average length of a price spell: both across the population of all price spells (corresponding to $\bar{d}$ in equation 6) and also across trajectories, where a mean duration is calculated for each trajectory and then the average is taken across trajectories (corresponding to $\bar{T}$). There are many detailed empirical issues to do with weighting, censoring and the introduction of the Euro which we ignore here. However, we can find the direct estimates of average durations taken across firms (all durations are in months) and contracts:

- Italy[11]: $\bar{d} = 8, \bar{T} = 13$.
- France[12]: $\bar{d} = 5.28, \bar{T} = 7.24$.
- Spain[13]: $\bar{d} = 6.2; \bar{T} = 14.7$.

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[10]See Dhyne et al (2005) for a summary of the IPN's findings.

[11]Veronese et al (2005), Table A2.

[12]Baudry et al (2004) page 16. Note, the estimate of $\bar{d}$ is only for unweigthed data. A trajectory in the French data is does not correspond to our concept of a complete trajectory over the whole period: the average length of trajectories is only 17 months. *Our* $\bar{T}$ is taken to be their $\bar{T}^W$: This will be an *underestimate*, becuase there are broken rather than complete trajectories.

These studies give the empirical distributions of contract lengths (price spells), but not across firms (trajectories). The distributions are very skewed: there are many short spells and a very long tale of long spells. The mean length of price-spells is 2-3 quarters. The empirical evidence shows that when you take the distribution across firms rather than across contracts, you get much longer average durations of 4-5 quarters (except for France) which is much more in line with the conventional wisdom and survey evidence that average durations are around 1 year.

2.3 Examples.

In this section we provide six examples of how our steady state indetities work. In the first column we state the reset probabilities (hazard rates) $\{\omega_i\}$; in the second and third the corresponding distribution of ages$\{\alpha_i^s\}$ and lifetimes $\{\alpha_i\}$ over firms, and in the fourth the distribution $\{\alpha_i^d\}$ over contracts (firms resetting prices). In the bottom row we compute the proportion of new contracts $\bar{\omega}$, the average age of contracts $\bar{s}$ and the average lifetime $\bar{T}$ across firms in steady state, and $\bar{d}$ the average lifetime across contracts[14].

Example 1

$$\begin{array}{llll}
\omega_1 = \frac{9}{10} & \alpha_1^s = \frac{37}{40} & \alpha_1 = \frac{9}{10} & \alpha_1^d = \frac{36}{37} \\
\omega_2 = 0 & \alpha_2^s = \frac{1}{40} & \alpha_2 = 0 & \alpha_2^d = 0 \\
\omega_3 = 0 & \alpha_3^s = \frac{1}{40} & \alpha_3 = 0 & \alpha_3^d = 0 \\
\omega_4 = 1 & \alpha_4^s = \frac{1}{40} & \alpha_4 = \frac{1}{10} & \alpha_4^d = \frac{1}{37} \\
\bar{\omega} = \frac{37}{40} & \bar{s} = \frac{23}{20} & \bar{T} = \frac{13}{10} & \bar{d} = \frac{40}{37}
\end{array}$$

In this example, there are two lengths of contracts: 90% are 1 period and 10% 4 periods. Note that $\bar{d} < \bar{s} < \bar{T}$: because of the proliferation of short contracts, the mean *lifetime* across contracts is even less than the average *age* across firms (in all the other examples, $\bar{d} > \bar{s}$).

[13] Alverez and Hernando (2004). $\bar{T}$ is taken from Table 6.1 Panel C. There is no direct measure of $\bar{d}$ using the CPI weights (Panel A.gives the unweighted mean). The value quoted is derived from the inverse of the reset frequency for each sector aggregated using the CPI weights (page 13).

[14] Note that the value of $\bar{\omega}$ is computed directly from the hazards ω_i; likewise $\bar{d}$ from α_i^d. The fact that $\bar{\omega} = \alpha_1^s = \bar{d}^{-1}$ reflects the consistency of the identities.

Example 2

$$\begin{array}{llll} \omega_1 = \frac{1}{4} & \alpha_1^s = \frac{8}{17} & \alpha_1 = \frac{2}{17} & \alpha_1^d = \frac{1}{4} \\ \omega_2 = \frac{1}{2} & \alpha_2^s = \frac{6}{17} & \alpha_2 = \frac{6}{17} & \alpha_2^d = \frac{3}{8} \\ \omega_3 = 1 & \alpha_3^s = \frac{3}{17} & \alpha_3 = \frac{9}{17} & \alpha_3^d = \frac{3}{8} \\ \bar{\omega} = \frac{8}{17} & \bar{s} = \frac{29}{17} & \bar{T} = \frac{41}{17} & \bar{d} = \frac{17}{8} \end{array}$$

This example has a gently rising reset probability, with the shares of completed contracts across firms increasing with length of contract, as do the shares across contracts.

Example 3

$$\begin{array}{llll} \omega_1 = \frac{1}{4} & \alpha_1^s = \frac{32}{71} & \alpha_1 = \frac{8}{71} & \alpha_1^d = \frac{1}{4} \\ \omega_2 = \frac{1}{2} & \alpha_2^s = \frac{24}{71} & \alpha_2 = \frac{24}{71} & \alpha_2^d = \frac{3}{8} \\ \omega_3 = \frac{3}{4} & \alpha_3^s = \frac{12}{71} & \alpha_3 = \frac{27}{71} & \alpha_3^d = \frac{27}{96} \\ \omega_4 = 1 & \alpha_4^s = \frac{3}{71} & \alpha_4 = \frac{12}{71} & \alpha_4^d = \frac{3}{32} \\ \bar{\omega} = \frac{32}{71} & \bar{s} = \frac{128}{71} & \bar{T} = \frac{185}{71} & \bar{d} = \frac{71}{32} \end{array}$$

This is similar to example 2, with a rising hazard over four periods. The shares across firms and contracts both peak at period 3 with a small 4-period share.

Example 4: Simple Taylor 4.

$$\begin{array}{lll} \omega_1 = 0 & \alpha_1^s = \frac{1}{4} & \alpha_1 = \alpha_1^d = 0 \\ \omega_2 = 0 & \alpha_2^s = \frac{1}{4} & \alpha_2 = \alpha_2^d = 0 \\ \omega_3 = 0 & \alpha_3^s = \frac{1}{4} & \alpha_3 = \alpha_3^d = 0 \\ \omega_4 = 1 & \alpha_4^s = \frac{1}{4} & \alpha_4 = \alpha_4^d = 1 \\ \bar{\omega} = \frac{1}{4} & \bar{s} = \frac{5}{2} & \bar{T} = \bar{d} = 4 \end{array}$$

A simple lesson can be derived from example 4. When all contracts have the same length, the distribution across contracts is the same as the distribution across firms.

Example 5: Taylor's US economy We can now consider an example starting from an empirical distribution of completed contract lengths we can derive the corresponding GC. Taylor's US economy represents the estimated distribution of completed contract lengths[15] (in quarters) in the third column. We can represent this in terms of the distribution of

[15]In fact, in Taylor (1993), the ages are estimated but not reported. In Table 2.2 page 48 the second column we believe to be the distribution $\{\alpha_i\}$ although it is reported as $\{a_i^d\}$: in the text, it says that "contract lengths in the three to four quarter range appear

ages and duration dependent hazard rates (both to 4 Decimal places), distribution over contracts and the resultant averages.

$\omega_1 = 0.2017$	$\alpha_1^s = 0.3470$	$\alpha_1 = 0.07$	$\alpha_1^d = 0.2017$
$\omega_2 = 0.3430$	$\alpha_2^s = 0.2770$	$\alpha_2 = 0.19$	$\alpha_2^d = 0.2738$
$\omega_3 = 0.4213$	$\alpha_3^s = 0.1820$	$\alpha_3 = 0.23$	$\alpha_3^d = 0.1825$
$\omega_4 = 0.4986$	$\alpha_4^s = 0.1052$	$\alpha_4 = 0.21$	$\alpha_4^d = 0.1513$
$\omega_5 = 0.5682$	$\alpha_5^s = 0.0528$	$\alpha_5 = 0.15$	$\alpha_5^d = 0.0865$
$\omega_6 = 0.5849$	$\alpha_6^s = 0.0228$	$\alpha_6 = 0.08$	$\alpha_6^d = 0.0384$
$\omega_7 = 0.6038$	$\alpha_7^s = 0.0095$	$\alpha_7 = 0.04$	$\alpha_7^d = 0.0165$
$\omega_8 = 1$	$\alpha_8^s = 0.0037$	$\alpha_8 = 0.03$	$\alpha_8^d = 0.0108$
$\bar{\omega} = 0.3470$	$\bar{s} = 2.365$	$\bar{T} = 3.730$	$\bar{d} = 2.8818$

It is interesting to note that here, unlike examples 1-4, we can really see the difference between the distribution across contracts and across firms: in the distribution across contracts durations 1 and 2 are really boosted - we see a lot of shorter contracts. All the other durations are reduced, and in particular the longer contract lengths are much less common in the distribution across contracts and across firms. The resultant mean duration is 77% of the mean across firms.

Example 6: Simple Calvo The Calvo model most naturally relates to the hazard rate approach to viewing the steady state distribution of durations. The simple Calvo model has a constant reset probability ω (the hazard rate) in any period that the firm will be able to review and if so desired reset its price. This reset probability is exogenous and does not depend on how long the current price has been in place. We can think about a sequence of uninterrupted periods without any review as the "contract length". The distribution of ages of contracts is

$$\alpha_s = \omega \left(1-\omega\right)^{s-1} : \ s = 1...\infty$$

which has mean $\bar{s} = \sum_{s=1}^{\infty} \alpha_s.s = \omega^{-1}$. Applying Proposition 1(a) gives us the steady-state distribution of completed contract lengths i across firms:

$$\alpha_i = \omega^2 i \left(1-\omega\right)^{i-1} : \ i = 1...\infty \tag{7}$$

to predominate". The third column which is reported as $\{\alpha_i\}$ is monotonic so may be ages. We have not been able to find an interpretation of Table 2.2 which is consistent with the steady state identities in this paper.

which has mean $\bar{T} = 2\omega^{-1} - 1$ (see Dixon and Kara 2006). Note that for the simple Calvo model, the distribution of ages is the same as the distribution across contracts: substituting (7) into (5) yields $\alpha_i^s = \alpha_i^d$ $i = 1...\infty$, so that the mean *age* of contracts across firms equals the mean *lifetime* across new contracts and is the reciprocal of the reset probability. We illustrate the simple Calvo model with $\omega = 0.25$.

$\omega_1 = 0.25$	$\alpha_1^s = 0.25$	$\alpha_1 = 0.0625$	$\alpha_1^d = 0.25$
$\omega_2 = 0.25$	$\alpha_2^s = 0.1875$	$\alpha_2 = 0.09375$	$\alpha_2^d = 0.1875$
$\omega_3 = 0.25$	$\alpha_3^s = 0.1406$	$\alpha_3 = 0.10546875$	$\alpha_3^d = 0.1406$
$\omega_4 = 0.25$	$\alpha_4^s = 0.1052$	$\alpha_4 = 0.10546875$	$\alpha_4^d = 0.1052$
$\omega_i = 0.25$	$\alpha_i^s = 0.25\,(0.75)^{i-1}$	$\alpha_i = (0.25)^2\, i\,(0.75)^{i-1}$	$\alpha_i^d = \alpha_i^s$
$\bar{\omega} = 0.25$	$\bar{s} = 4$	$\bar{T} = 7$	$\bar{d} = 4$

3 Pricing Models with steady state distributions of durations across firms.

Having derived a unified framework for understanding the set of all possible steady state distributions of durations across firms, we can now see how this can be used to understand commonly used models of pricing behaviour based on generalisations of Taylor and Calvo.

The Generalised Taylor Economy GTE Using the concept of the Generalised Taylor economy GTE developed in Dixon and Kara (2005a), any steady-state distribution of completed durations across firms $\boldsymbol{\alpha} \in \Delta^{F-1}$can be represented by the GTE with the sector shares given by $\boldsymbol{\alpha} \in \Delta^{F-1}$: $GTE\,(\boldsymbol{\alpha})$. In each sector i there is an $i-$period Taylor contract, with i cohorts of equal size (since we are considering only uniform $GTEs$). The sector share is given by α_i. Since the cohorts are of equal size and there as many cohorts as periods, there are $\alpha_i.i^{-1}$ contracts renewed each period in sector i. This is exactly as required in a steady-state. Hence the set of all possible $GTEs$ is equivalent to the set of all possible steady-state distributions of durations. It is simple to verify that the age-distribution in a GTE is given by (4). If we want to know how many contracts are at aged j periods, we look at sectors with lifetimes at least as large as j, $i = j...F$. In each sector

i, there is is a cohort of size $\alpha_i.i^{-1}$ which set its price j periods ago. We simply sum over all sectors $i \geq j$ to get (4). The simple Taylor economy (ST) is a special case where there is only one sector and one contract length.

The Generalised Calvo model (GC)**.** The representation of the steady-state distribution by hazard rates suggests generalising the Calvo model to allow for the reset probability (hazard) to vary with the age of the contract (a duration dependent hazard rate), as in Wolman (1999),. This we will denote the *Generalised Calvo Model GC*. A GC is defined by a sequence of reset probabilities: as in the previous section this can be represented by any $\omega \in [0,1)^{F-1}$. From observation 1, given any possible GC there is a unique age profile $\alpha^s \in \Delta_M^{F-1}$ corresponding to it and a unique distribution of completed contract lengths from Proposition 1. Again, Proposition1, if we have a distribution of completed contract lengths, there is a unique GC which corresponds to it. Thus, the two approaches to modelling pricing: the GTE and the GC are comprehensive and coextensive, both being consistent with any steady-state distribution of durations[16].

The Multiple Calvo Model (MC)**.** In this approach, the economy is seen of as comprising several sectors, each with a simple Calvo process (i.e. a sector specific hazard/reset probability). Alvarez et al (2005) argue that an aggregate hazard rate declines over time and that this can be attributed to the heterogeneity of hazard rates. We can define a multiple Calvo process MC as $MC(\bar{\boldsymbol{\omega}}, \boldsymbol{\beta})$ where $\bar{\boldsymbol{\omega}} \in (0,1]^n$ gives a sector specific hazard rate[17] $\bar{\omega}_k$ for each sector $k = 1, ...n$ and $\boldsymbol{\beta} \in \boldsymbol{\Delta}^{n-1}$ is the vector of shares β_k.

We can thus see that all of the Above models generate a steady state distribution of durations. We can ask which pricing models do not yield a steady state. The important thing to note about the above models is that

[16]Note that an alternative parameterization of the duration dependent hazard rate model is to specify not the hazard rate at each duration, but rather the probability of the completed contract length at birth (see for example Guerrieri 2004). The probability at birth f_i of a contract lasting exactly i periods is simply the probability it survives to period i and then resets at $i : f_i = \Omega_i \omega_i$.

[17]The notation here should not be confused: the substrcripts k are sectoral: none of the sectoral calvo reset probabiltities are duration dependent in the MC model.

the distrbution of contract lengths does not depend in any way upon the state of the economy: it is essentially the result of an an exogenous process which terminates contracts. Thus even though the firm or the economy are not in steady state, the distribtion of durations is invariant. State dependent or menu cost models do not have this property (for example Dotsey et al 1999). When the economy is in steady state they do posses a steady state distribution of durations. However, the state of the economy affects the duration of contracts, which means that when the economy is out of steady state (due to an exogenous shock of some kind), durations of contracts may also deviate from the steady state.

3.1 Modelling a Multiple Calvo economy as a Generalised Taylor and Generalised Calvo economy.

Having identified three different pricing models, we can use the unified framework to show how we can move betweeen them. The example we will use is the MC model: we will show how to model this as a GTE and a GC with the same steady state distribution of durations across firms. So, let us take as the starting point the multiple-Calvo porcess $MC(\bar{\boldsymbol{\omega}}, \boldsymbol{\beta})$. To model this as a GTE, we can take the distribution of duration within each sector: let α_{ki} be the proportion of i period contracts in the k, from (7) we have:

$$\alpha_{ki} = \bar{\omega}_k i \left(1 - \bar{\omega}_k\right)^{i-1}$$

The proportion of i period contracts across the whole economy, α_i is obtained by summing accross sectors using the weights β_k

$$\alpha_i = \sum_{k=1}^{n} \beta_k \alpha_{ki} \tag{8}$$

Hence we can represent $MC(\bar{\boldsymbol{\omega}}, \boldsymbol{\beta})$ by $GTE(\boldsymbol{\alpha})$ using (8).

From this distribution of completed durations, we can construct the corresponding GC from Proposition 1(b). The flow of new contracts is $\bar{\omega}$ and the aggregate period i hazard is ω_i which we can either define in terms of the distribution of lifetimes or the underlying sectoral reset probabilities:

$$\bar{\omega} = \sum_{i=1}^{\infty} \frac{\alpha_i}{i} = \sum_{i=1}^{\infty} \frac{\sum_{k=1}^{n} \beta_k \alpha_{ki}}{i} = \sum_{i=1}^{\infty} \sum_{k=1}^{n} \beta_k \bar{\omega}_k \left(1 - \bar{\omega}_k\right)^{i-1}$$

$$\omega_i = \frac{\alpha_i}{i} \left(\sum_{j=0}^{\infty} \frac{a_{i+j}}{i+j} \right)^{-1} = \frac{\sum_{k=1}^{n} \beta_k \bar{\omega}_k \left(1 - \bar{\omega}_k\right)^{i-1}}{\sum_{j=0}^{\infty} \sum_{k=1}^{n} \beta_k \bar{\omega}_k \left(1 - \bar{\omega}_k\right)^{i+j-1}}$$

Proposition 2: The aggregate GC model corresponding to MC model has a declining hazard rate. In the limit as $i \rightarrow \infty$, the hazard rate in the GC tends to the lowest hazard rate in the MC.

Clearly, the aggregate hazard in the GC corresponding to an MC is decreasing over time: $\omega_i > \omega_{i+1}$. The way to understand this is that sectors with higher $\bar{\omega}_k$ tend to change contract sooner. So, for a given cohort, the relative share of sectors with higher $\bar{\omega}_k$ tends to go down. At any duration i, the share of type k contracts increases if the reset probability is below average or decreases if it is above average. The reset probability gradually declines and asymptotically reaches the lowest reset probability. In the long-run, the type with the lowest reset probability comes to dominate and the GC tends to this lowest vale.

4 The Typology of Contracts and Aggregation.

In terms of contract structure, we can say that the following relationships hold:

- $GC = GTE = SS$. The set of all possible steady state distributions of durations is equivalent to the set of all possible GTEs and the set of all possible GCs.
- $C \subset MC \subset GC$. The set of distributions generated by the Simple Calvo is a special case of the set generated by MC which is a special case of GC.
- $ST \subset GTE = GC$ Simple Taylor is a special case of GTE, and hence also of GC.

- $ST \cap MC = \varnothing$. Simple Taylor contracts are a special case of GC, but not of MC.

Figure 1: The typology of Contracts

This is depicted in Fig 1. The GC and the GTE are coextensive, being the set of all possible steady-state distributions (Proposition 1). The Simple calvo C (one reset probability) is a strict subset of the Multiple Calvo process MC which is a strict subset of the GC. The simple Taylor ST and the MC are disjoint. The ST is a strict subset of the GTE. The size of the distributions is reflected by the Figure: ST has elements corresponding to the set of integers and is represented by a few dots; Calvo is represented by the unit interval; MC by the unit interval squared.

We can now ask the question: if we aggregate over two contract structures, what is the type of contract structure that results? This is an important question: if we believe that the economy is heterogenous, we should not represent it with a contract type which is not closed under aggregation. We can think of this in terms of giving each contract structure a strictly positive proportion of the total set of contracts; for example 50%. We can define the ST in terms of contract length, under the assumption that each cohort is of equal size.

$$ST(k) + ST(j) = GTE\left((0.5, 0.5), (k, j)\right)$$

Clearly, if we aggregate over Standard Taylor contracts with different contract lengths $k > j$, we no longer have a Standard Taylor contract but a GTE.

Similarly, if we aggregate over simple $Calvo$ contracts with different reset probabilities, we do not get a C contract but a multiple Calvo MC:

$$C(\omega_1) + C(\omega_2) = MC\left((0.5, 0.5), (\omega_1, \omega_2)\right)$$

By definition, If we aggregate over MCs, we still have an MC. We can say that a type of contract structure is closed under the operation of aggregation if we aggregate two different contracts of that type and the resultant contract structure is also of the same type. Clearly, neither the ST or C are closed under aggregation. However, MC, GC, and GTE are all closed under aggregation:

Observation MC, GC, and GTE are all closed under aggregation.

Consider the case of GTE. Suppose we have two $GTEs$ with maximum contract lengths F_1 and F_2 respectively and w.l.o.g. $F_1 \leq F_2$. The corresponding vector of sector shares is then $\boldsymbol{\alpha}_j \in \Delta^{F_2-1}$, $j = 1, 2$ where we set $\alpha_{1i} = 0$ for $i > F_1$. If we combine the two $GTEs$, we get another GTE with the sector shares being the average of the other two.

$$GTE(\boldsymbol{\alpha}_1) + GTE(\boldsymbol{\alpha}_2) = GTE\left(\frac{1}{2}\boldsymbol{\alpha}_1 + \frac{1}{2}\boldsymbol{\alpha}_2\right)$$

Hence $GTEs$ are closed under aggregation. Similarly, since GCs can be represented as equivalent $GTEs$, the closure of $GTEs$ implies the closure of GC under aggregation.MC are closed, since we can simply combine the different ω_i from each MC and reweight on a 50-50 basis. We have illustrated the proof using two distributions with a 50-50 combination. The idea obviously generalises to a convex combination of any number of MC, GTE and GCs.

The importance of this observation is that if we really do believe that contract structures are heterogeneous, we should use contract types that are closed under aggregation. The simple Calvo and Taylor models are only applicable if there is one type of contract and no heterogeneity in the economy. If we believe the Calvo model, but that reset probabilities are heterogenous across price or wage setters, then the MC makes sense. If we believe that the Calvo model is not a good one, then the GC or GTE is appropriate.

5 The Forward Lookingness of pricing rules.

We have developed a general framework for understanding steady-state distributions of durations across firms and how they are related in terms of pricing models. In this section we consider how pricing models differ when we control for the distribution of durations (requiring the steady state distributions to be the same) in terms of the "Forward Lookingness" (FL) of the pricing rules. A pricing rule uses data from the present and future in order to determine the optimal price. In a log-lineraised form, this gives the current price as a linear function of data from each date ahead. The forward lookingness of a pricing rule takes the weights (normalised to unity) in the linearised decision rule and is the resultant average over the dates ahead. This simple measure capures the extent to which the future influences the pricing decision, and is applicable across all pricing rules.[18]. This concept was introduced in Dixon and Kara (2005) to compare the simple Calvo model and the equivalent GTE (named the Calvo-GTE). The Calvo-GTE is a model with exactly the same distribution of completed contract lengths as in the Calvo model, as given by ($7b$). We now generalise this to the GC framework. In this paper we ignore discounting, since it applies to all pricing rules: however, whilst it would be simple to generalise the formulae to allow for discounting, the no-discount case allows us to understand the differences more clearly.

Let the optimal price (or target variable in general) in period $t+s$ be P^*_{t+s} so that the future information influencing the pricing decision at t is $\left\{P^*_{t+s}\right\}_{s=0}^{\infty}$. In a GC with $\boldsymbol{\omega} \in [0,1)^{F-1}$, the proportion of firms resetting price at time t is $\bar{\omega}$ and they all set the same reset price X_t^{GC}. Ignoring discounting, the reset wage and forward lookingness[19] FL are:

$$X_t^{GC} = \bar{\omega} \sum_{j=0}^{F-1} \Omega_j P^*_{t+j-1} = \sum_{j=0}^{F-1} \alpha_j^s P^*_{t+j-1} \tag{9}$$

The weights in the GC are the distribution of ages α_j^s. This means that the forward lookingness of the GC is simply the average age of the distribution:

$$FL^{GC} = \bar{\omega} \sum_{s=1}^{F-1} s\Omega_s = s\alpha_j^s = \bar{s} \tag{10}$$

In the corresponding GTE there are F sectors, with contract lengths $i = 1...F$ with corresponding sector shares given by (2):

$$\alpha_i = i.\omega_i \alpha_i^s$$

[18]Note that Forward Lookingness is not in general equal to the expected duration of the contract when the price is set (life expectation at birth). They are equal in the simple Calvo model because it has the special property that life expectancy at birth equals the average age. For example, in the simple Taylor model without discounting $FL = \bar{s}$, but life expectation at birth equals the full length of the contract (average completed contract length $\bar{T}$).

[19]Note that Forward Lookingness is not in general equal to the expected duration of the contract when the price is set (life expectation at birth). They are equal in the simple Calvo model because it has the special property that life expectancy at birth equals the average age. For example, in the simple Taylor model, $FL = \bar{s}$, but life expectation at birth equals the full length of the contract (average completed contract length $\bar{T}$).

In each sector i, a proportion i^{-1} reset their prices every period: hence a total of $\bar{\omega}$ reset their prices, since from Proposition 1:

$$\bar{\omega} = \sum_{i=1}^{F} \frac{\alpha_i}{i} \tag{11}$$

The important thing to note about (11) is that the longer contract lengths are under-represented amongst resetters relative to the population, since they reset prices less often. This means that the forward lookingness of those making the pricing decision is much less than the average contract length.

The reset price in sector i will be

$$X_{it} = \frac{1}{i} \sum_{j=0}^{i-1} P^*_{t+j}$$

The average reset price *across those resetting* price is thus

$$\begin{aligned} X_t^{GTE} &= \frac{1}{\bar{\omega}} \sum_{i=1}^{F} \frac{\alpha_i}{i} X_{it} = \frac{1}{\bar{\omega}} \sum_{i=1}^{F} \frac{\alpha_i}{i^2} \sum_{j=0}^{i-1} P^*_{t+j} \\ &= \frac{1}{\bar{\omega}} \sum_{i=1}^{F} \frac{\omega_i \alpha_i^s}{i} \sum_{j=0}^{i-1} P^*_{t+j} \\ &= \sum_{j=1}^{F} b_s P^*_{t+j-1} \end{aligned}$$

where the weights on future information b_s can be written

$$b_j = \sum_{i=j}^{F} \frac{\alpha_i}{\bar{\omega} i^2} = \sum_{i=j}^{F} \frac{\omega_i \alpha_i^s}{\bar{\omega} i} \tag{12}$$

Hence the forward lookingness of the GTE is

$$FL^{GTE} = \sum_{j=1}^{F} j b_j$$

It is illuminating to write the GTE weights in terms of the GC weights. In the GTE, each price setter knows the exact length of the contact: hence when setting price it ignores what happens after the end of the contract. In contrast, in the GC the price-setter is uncertain of the contract length and must always consider the possibility of lasting until the longest duration F. As we identified in Dixon and Kara (2005), this results in the fact that comparing the GTE to the GC weights, weight is "passed back" from longer durations to shorter making the GTE more myopic:

$$b_j = \frac{\omega_j}{\bar{\omega}}\alpha_j^s - \frac{j}{j+1}\frac{\omega_j}{\bar{\omega}}\alpha_j^s + \sum_{i=j+1}^{\infty}\frac{\omega_i}{\bar{\omega}}\frac{\alpha_i^s}{i} \tag{13}$$

There are three components of b_j in (13): the corresponding α_j^s, the weight passed back to more recent dates $i < j$ and thirdly the weight it receives from longer contracts which include . To translate from α_j^s to b_j, you need to correct by a factor of $\omega_j/\bar{\omega}$ for each duration j: in the simple Calvo model this is unity and the equation reverts to Dixon and Kara (2005). This means that the $b_j s$ put a greater weight on the immediate future and less on the more distant future than the corresponding GC. We can see this if we look at the *cumulative* weights: looking at the sum of weights up to q periods ahead, the sum of $b_j s$ is the sum of $C_j s$ plus the weights passed back from the periods in the future to each of the b_j where $j \leq q$.

$$\sum_{j=1}^{q} b_j = \sum_{j=1}^{q}\frac{\omega_j}{\bar{\omega}}\alpha_j + \sum_{k=0}^{q}\sum_{i=k}^{\infty}\frac{\omega_i}{\bar{\omega}}\frac{\alpha_i}{i+1}$$

The intuition behind this result is the following. In the GC, all firms are uncertain about how long the price they set will last: all firms have to take into account the optimal price for all F periods. This results in the optimal weights for each future period being the corresponding age shares. In the GTE, however, firms only worry about the prices that cover the known contract length. Thus firms with perfectly flexible prices can ignore the future when they set prices. Firms with i period contracts just look i periods ahead. The only people to worry about the optimal price $F-1$ periods ahead are the firms with F period contracts. However, they also have to worry about all the prices along the way. Thus, when we consider (13), the bit "passed back" reflects the fact that the i period contracts have to take into account the periods up to i: the bit received from longer contracts is the corresponding bit they pass back to period i.

We can illustrate the differences in forward lookingness using the some of the examples we considered before in section 3.1. Since the GC weights are just the age-shares α_i^s and were given previously as fractions in section 2.2, we state them here as decimals to 4 decimal places. The b_j coefficients are give as both exact fractions and decimals.

Example 1

$$\begin{array}{rr} \alpha_1^s = 0.925 & b_1 = 0.9521 = \frac{457}{480} \\ \alpha_2^s = 0.025 & b_2 = 0.0271 = \frac{13}{480} \\ \alpha_3^s = 0.025 & b_3 = 0.0146 = \frac{7}{480} \\ \alpha_4^s = 0.025 & b_4 = 0.0063 = \frac{1}{160} \\ FL^{GC} = 1.15 & FL^{GTE} = \frac{486}{480} = 1.075 \end{array}$$

Example 2.

$$\begin{array}{rr} \alpha_1^s = 0.4706 & b_1 = \frac{9}{16} = 0.5625 \\ \alpha_2^s = 0.3530 & b_2 = \frac{5}{16} = 0.3125 \\ \alpha_3^s = 0.1765 & b_3 = \frac{2}{16} = 0.125 \\ FL^{GC} = 1.706 & FL^{GTE} = \frac{25}{16} = 1.5625 \end{array}$$

Example 3

$$\begin{array}{rr} \alpha_1^s = 0.4507 & b_1 = \frac{71}{128} = 0.5547 \\ \alpha_2^s = 0.3380 & b_2 = \frac{39}{128} = 0.3047 \\ \alpha_3^s = 0.1690 & b_3 = \frac{15}{128} = 0.1172 \\ \alpha_4^s = 0.0423 & b_4 = \frac{3}{128} = 0.0234 \\ FL^{GC} = 1.803 & FL^{GTE} = 1.609 \end{array}$$

Clearly, in all three examples the GTE corresponding to a GC puts a much greater weight on the current period and less on the subsequent periods, resulting in a less forward looking pricing decision. In example 1, the weight is still greater on the second period, but falls off rapidly.

Lastly, we can consider the case of a MC process. In this case, the forward lookingness of the MC is simply the average of the Forward lookingnesses of the individual Calvo processes weighted by sector shares β_k

$$FL^{MC} = \sum_{k=1}^{n} \beta_k \bar{\omega}_k = \sum_{j=1}^{k} \beta_k \bar{s}_k = \bar{s}$$

where $\bar{s}_k$ is the average age in steady state in the $k-$sector, and $\bar{s}$ the average age in the population. Since by construction the MC and the equivalent GC have the same distribution of ages in steady state and hence average age in the population, the two different constructions have the same forward-lookingness, $FL^{GC} = FL^{MC}$. Furthermore, we can see that the average reset price at time t will be equal.

In the MC, there will be different reset prices, one for each $\bar{\omega}_k$. Hence the reset price of firms with hazard $\bar{\omega}_k$ is:

$$X_{kt}^{MC} = \sum_{j=1}^{\infty} \alpha_{kj}^{s} P_{t+j-1}^{*}$$

where $\left\{\alpha_{kj}^{s}\right\}_{j=1}^{\infty}$ is the steady-state age-distribution for those with hazard $\bar{\omega}_k$. The average reset price is then

$$\begin{aligned} X_t^{MC} &= \sum_{k=1}^{n} \beta_i X_{kt}^{MC} = \sum_{k=1}^{n} \sum_{j=1}^{\infty} \beta_k \alpha_{kj}^{s} P_{t+j-1}^{*} \\ &= \sum_{j=1}^{\infty} \alpha_j^{s} P_{t+j-1}^{*} \end{aligned}$$

since $\sum_{k=1}^{n} \beta_k \alpha_{kj}^{s} = \alpha_j^{s}$.

Hence the average reset price in the MC is the same as the equivalent GC setting. This means that when modelling an economy with heterogeneous Calvo contracts as in the MC model, it may well be the most parsimonious to use the GC framework. The degree of forward lookingness and the average reset price are the same. The only difference is that in MC there are as many reset prices as hazard rates, whereas in the GC there is only one.reset price in any one period.

6 Price Data: an application to the Bils-Klenow Data set.

In this section, we apply our theoretical framework to the Bils-Klenow Data set (Bils and Klenow 1994). This data is the micro-price data collected monthly for the US CPI over the period 1995-7. The BK data covers 350 categories of commodities comprising 68.9% of total consumer expenditure. They focus on the proportion of prices that change in a month in each category (sector). They then derive the distribution of durations across contracts on the assumption that there is a sector specific Calvo reset probability in continuous time. As we have argued, we belive that the distribution across contracts is not the right way to quantify price-stickiness.

In this section I use the BK data to construct the distribution of contract lengths across firms. Each sector has a sector-specific average proportion of firms resetting their price per month over the period covered. I interpret this as a Calvo reset probability in *discrete* time[20]. We adopt the discrete time approach in order to be consistent with the pricing models which are in discrete time. The first approach we adopt is to model this as a Multiple Calvo process $BK - MC$. The second is to model the resulting distribution across all sectors. Within each sector we have the Calvo distribution of contract lengths as derived in Dixon and Kara (2006a): using the sectoral weights we can then aggregate across all sectors. This gives us the following distribution of contract lengths depicted in Figure 2:

Fig 2: the BK Distribution of Contract lengths Across Firms

Note that the mean of 4.4 quarters is larger than is reported in BK. This is because we are looking at the mean duration across firms rather than contracts and hence we are more likely to observe longer contracts. With the aggregate distribution of contract lengths we can model this as either a GTE or a GC as well as an MC. We therefore have three different pricing models of the same distribution of contract lengths derived from the BK dataset.

6.1 Pricing Models Compared.

We will see how the different models of pricing differ in terms of their impulse-response. We adopt the model of price or wage setting developed in Dixon and Kara (2005a, 2006): the details are set out in the appendix. We consider two monetary policy shocks: in the first case there is a one off permanent shock in the level of the money supply; in the second an autoregressive

[20]The use of continuous time leads to a lower expected expected duration at birth. If the proportion resetting price is $\bar{\omega}$, the expected duration at birth is $-1/In(1-\bar{\omega})$. This is less than the discrete time expectation $1/\bar{\omega}$, partly reflecting the fact that the price might change more than once in a given period. The difference gets proportinatley larger as $\bar{\omega}$ gets larger. When $\bar{\omega} = 0.8$ the discrete time estimator is over twice the continuous time estimator. The analysis in this paper is in discrete time becuase that is how the pricing models are employed in the literature, and also it provides spreadsheet simplicity and trnasparency.

process. Expressed as log deviations form steady state, money follows the process

$$\begin{aligned} m_t &= m_{t-1} + \varepsilon_t \\ \varepsilon_t &= \nu\varepsilon_{t-1} + \xi_t \end{aligned}$$

where ξ_t is a white noise error term. We consider the case of $\nu = 0$ and the autoregressive case $\nu = 0.5$. The other key parameter γ captures the sensitivity to the flexible price to output. The optimal flexible price at period t in any sector p_t^* is given by

$$p_t^* = p_t + \gamma y_t$$

where (p_t, y_t) are aggregate price and output (all in log-deviation form). We allow for two values of $\gamma = \{0.01, 0.2\}$: a high one and a low one as discussed in Dixon and Kara (2006b).

Fig 3: Responses to a one-off monetary Shock.

In Figure 3, we depict the responses of output, the reset price, the general price level and inflation to a one-off shock with $\gamma = 0.2$. Looking at all the graphs, it is striking that the three models of pricing have fairly similar impulse-responses: none of them are far apart. However, in all cases the MC and the GC are close together and the GTE is farther away, particularly towards the end. To understand this, we can look at the IR for the average reset price and the general price level. In the GTE case, the reset price rises less on impact than the MC or GC. This reflects the greater myopia: those cohorts resetting prices look less far ahead on average, so that they do not raise prices as much as in the MC or GC case. At about 10 months however, the situation is reversed: the GTE reset price exceeds the MC and GC case: whilst the latter are slowing down price increases in anticipation of the approaching steady state, the GTE maintains momentum for longer. This comparative myopia of the GTE explains why the output response starts off above both the MC and GC, but ends up after 15 months below both.

Fig 4: Serial Correlation in Monetary growth $\nu = 0.5$

In Figure 4 we consider the autoregressive monetary policy shock and concentrate on the IR for output and inflation for both the high and the low values of γ. We find that there is now a more radical difference between the GTE and the other two models. If we look at inflation we see that there is a hump shape: the peak impact on inflation appears after the initial monetary shock: with the high value of γ it happens at 3 months: with the low value at around 20 months. Both the MC and the GC are not hump shaped. This reflects the finding in Dixon and Kara 2006b that the Calvo model does not capture the characteristic "hump shaped" response indicated by empirical VARS. This feature appears tp be shared by its generalisations MC and GC.

This simple example of the IR of major variables shows how different models of pricing can yield different patterns of behaviour even though the distribution of contract lengths are exactly the same. Partly this is due to different degrees of forward lookingness. The MC and the GC do differ slightly, but are quite close, which reflects the fact that they have the same forward lookingness. It suggests that since the GC is computationally much simpler (you only have to model one pricing decision for all firms resetting price, rather than one for each sector), this model might be preferred to the MC.

6.2 Alternative Interpretations of the BK Data set.

The previous analysis was based on assuming that the true distribution within each sector is generated by the sector specific discrete time Calvo distribution. However, this is just one hypothesis about the underlying distribution of contract lengths generating the proportion of firms resetting their price per month. Let us look at the class of GTEs that are consistent with a particular reset proportion. Again, let us consider the set of GTEs with maximum contract lengths F, $\boldsymbol{\alpha} \in \Delta^{F-1}$: we can define the subset which yield a particular reset proportion $\bar{\omega}$: define the mapping $A(\bar{\omega}) : [0,1] \to \Delta^{F-1}$

$$A(\bar{\omega}) = \left\{ \boldsymbol{\alpha} \in \Delta^{F-1} : \sum_{i=1}^{F} \frac{\alpha_i}{i} = \bar{\omega} \right\}$$

Note that since $A(\bar{\omega})$ is defined by a linear restriction on the sector shares $\boldsymbol{\alpha}$. We assume that $\bar{\omega} \geq F^{-1}$ in what follows, so that $A(\bar{\omega})$ is non-empty[21]. Hence $A(\bar{\omega}) \subset \Delta^{F-2}$ and $A(\bar{\omega})$ is closed and bounded. Note that if $\bar{\omega} = 1$, all

[21] This is a purely technical assumption. If we assumed a value $\bar{\omega} < F^{-1}$, then even if all contracts were at the maximum duration, there would be too many firms resetting prices. Since we are dealing with empirically relevant values, $\bar{\omega} > F^{-1}$ is automatically satisfied.

firms must reset their price every period, so that $\alpha_1 = 1$ and $A(\bar{\omega})$ collapses to a singleton. To avoid trivialities, we will restrict ourselves to $\bar{\omega} \in [F^{-1}, 1)$.

The average length of contracts is $\bar{T}(\boldsymbol{\alpha}) = \sum_{i=1}^{F} i.\alpha_i$. Let us consider the following problems: what is the lowest (highest) average contract length consistent with a particular reset proportion $\bar{\omega}$. Mathematically, we know that since $A(\bar{\omega})$ is non-empty, closed and bounded and $\bar{T}(\boldsymbol{\alpha})$ is continuous, both a maximum and a minimum will exist. Turning to the minimization problem first: we have to choose $\alpha \in \Delta^{F-1}$ to solve

$$\min \ \bar{T}(\boldsymbol{\alpha}) \quad s.t. \ \alpha \in A(\bar{\omega}) \tag{14}$$

Proposition 3 Let $\boldsymbol{\alpha}^* \in \Delta^{F-1}$ solve (14) to give the shortest average contract length $\bar{T}(\boldsymbol{\alpha})$.

(a) No more than two sectors i have values greater than zero

(b) If there are two sectors $\alpha_i > 0$, $\alpha_j > 0$ then will be consecutive integers ($|i - j| = 1$).

(b) There is one solution iff $\bar{\omega}^{-1} = k \in Z_+$. In this case, $\alpha_k = 1$.

We can sum up the proposition by saying that *the shortest average contract length consistent with a given proportion of resetters is the simplest GTE that can represent it.* It is either a pure simple Taylor, or an only slightly less simple GTE with two sectors of consecutive lengths. Note that whilst Proposition 3 is derived for a GTE, under the equivalence established by Proposition 1, it will also hold across all GCs. It is a distribution which minimises mean contract length, and this distribution can be seen as either a GTE or a GC.

We can also ask what is the *maximum* average contract length consistent with a proportion of resetters:

$$\max \ \bar{T}(\boldsymbol{\alpha}) \quad s.t. \ \alpha \in A(\bar{\omega})$$

Proposition 4 Given the longest contract duration F, the distribution of contracts that maximises the average length of contract subject to a given proportion of firms resetting price is

$$\begin{aligned} \alpha_F &= \frac{F}{F-1}(1-\bar{\omega}) \\ \alpha_1 &= \frac{F}{F-1}\bar{\omega} - \frac{1}{F-1} \end{aligned}$$

with $\alpha_i = 0$ for $i = 2...F-1$. The maximum average contract length is

$$\bar{T}^{\max} = F(1-\bar{\omega}) + 1$$

There will be one contract length with $\alpha_F = 1$ if $\bar{\omega} = F^{-1}$, and $\alpha_1 = 1$ if $\bar{\omega} = 1$. For all intermediate values, $\alpha_F.\alpha_1 > 0$.

This proposition implies that $\bar{T}^{\max} \to \infty$ as $F \to \infty$.

Let us return to the BK data set in the light of the preceding two propositions. First, we can ask the what is the shortest average contract length which is consistent with the BK data set. The BK data set gives us for each sector k the proportion of firms changing price: $\bar{\omega}_k$ in any month. Following Bils and Klenow themselves, it is natural to interpret this as a MC, that within each sector there is a Calvo process. This was the assumption we used to generate Figure 2. In terms of the mean contract length in the $BK - MC$, this is calculated as

$$\bar{T}^{MC} = \sum_{k=1}^{n} \alpha_k \frac{2-\bar{\omega}_k}{\bar{\omega}_k} = 4.4$$

Where durations are in Quarters unless otherwise specified. The minimum average contract length within each sector is simply $\bar{\omega}_k$, so that the minimum average contract length in the BK data is[22]:

$$\bar{T}^{\min} = \sum_{k=1}^{n} \alpha_k \frac{1}{\bar{\omega}_k} = 2.7$$

Clearly, for any data set like this (based on the proportion of firms changing price in a period), the shortest average contract length can be achieved using the Taylor model: using the Multiple Calvo yields an average duration nearly twice as long, with the linear relation:

$$\bar{T}^{MC} = 2\bar{T}^{\min} - 1.$$

The maximum average contract length is not revealed by the data set. There are some sectors with very low percentages of price changing: coin

[22] Note that this exceeds the level reported by Bils and Klenow! That is becuase they interpret the proportion as arising from a continuous time process. We are adopting the discrete time approach.

operated laundry has 1.2% of prices changing a month implying a *minimum*[23] mean contract length in that sector of 83 months (nearly 7 years). However, the nature of the BK data set says nothing about the distribution within the sectors, so there is no meaningful upper bound on the average length of contracts.

Lastly, let us consider what might happen if we aggregate over all sectors by taking the mean proportion of firms changing price,

$$\hat{\omega} = \sum_{k=1}^{n} \alpha_k \bar{\omega}_k$$

This gives us an estimate of average duration $\hat{T}$:

$$\hat{T} = \frac{2}{\hat{\omega}} - 1 = 2.47$$

Now, clearly, since contract length is a convex function of $\bar{\omega}_k$, by Jensen's inequality $\hat{T} \leq \bar{T}^{MC}$. If we use the actual BK data, we have $\hat{\omega} = 0.209$ per *month*, yielding the reported estimate of 2.47 quarters which is just over one half of the "true" MC value. This shows that aggregating over sectors in this way can be extremely misleading and will considerably underestimate the "true " value even if one believes the Calvo story[24]. However, the MC might not be the true model. Even with this degree of disaggregation, there may well be intra-sectoral variation. Ultimately, what is needed is the individual price data. Certainly, *using data of the proportions changing price in a period is of limited value and might not even get you into the right ballpark*. That is certainly what the BK data set tells us.

7 Conclusion

In this paper we have developed a consistent and comprehensive framework for analyzing different pricing models which generate steady state distributions of durations which can be used to understand dynamic pricing models. In particular, the d*istribution of completed contract lengths across firms* (DAF) is a key perspective which is fundamental to understanding and comparing different models. Any steady state distribution of durations can be

[23]Recall, we have the *lower* bound on F since $F \geq (\bar{\omega}_k)^{-1}$.

[24]This is also a finding emphasised on European data: see Dhyne E. et al (2005, 13).

looked at in terms of completed durations, which suggests it can be modelled as a GTE; it can also be thought of in terms of Hazard rates which suggests the GC approach. Both the GC and the GC are comprehensive: they can represent all possible steady states. Furthermore, they are closed under aggregation. Unlike the simple Calvo and Taylor models, they are consistent with heterogeneity in the economy.

Once we have controlled for the distribution of contract durations, we can compare different pricing models. The concept Forward Lookingness is useful in comparing the way different pricing rules put weights on the future periods. We find that the GC is less myopic than the corresponding GTE, echoing the finding of Dixon and Kara (2005a) comparing Calvo and the Calvo-GTE. We then illustrate this by using a standard macromodel using the Bils-Klenow data set, which we interpret (following Bils and Klenow) as an MC process. We see even though the distributions are identical, the three pricing models are different. The GC and MC are close to each other and had the same forward lookingness. The GTE is more myopic and has a different impulse-response in relation to a monetary shock. In particular, for particular parameterization, the GTE can display a hump-shaped inflation response, whereas the GC and MC never have a hump.

The analysis also has implications for how we interpret the data on the proportion of firms setting a price in a particular period. The minimum average length consistent with this is given by the simplest GTE. There is no reasonable upper bound unless we have an upper bound on the maximum contract length possible. Certainly, there are severe problems of aggregation which indicate that using such data might lead very inaccurate estimates of average contract length. The analysis also indicates that existing estimates of price-stickiness are biased downwards and that in reality prices are stickier than some have maintained.

Our analysis has looked at one type of wage or price setting behaviour: the contract consists in the setting of a nominal price or wage that persists throughout the contract. As we show in Dixon and Kara (2006b), other types of price and wage setting can be dealt with in this framework. For example, we can have Fischer contracts, where firms or unions set a trajectory of nominal prices over the life of the contract. This is essentially the approach taken by Mankiw and Reis (2002). There are other possibilities such as indexation during the contract once the initial level has been fixed. We can have any model of pricing so long as it is consistent with a steady state distribution of durations. The main class of pricing models that do not give a steady state distribution are of course the state-dependant pricing models (menu cost models), such as Dotsey, King and Wolman. (1999). Here the duration or contracts depends on the macroeconomic environment. However, this paper makes an attempt to improve our understanding of the steady state, which will in turn provide a firmer foundation for understanding non-steady state phenomena.

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9 Appendix.

9.0.1 Proposition 1(a)

Proof. From observation 1, the flow of new contracts is $\alpha_1^s = \bar{\omega}$ each period. To survive for exactly i periods, you have to survive to period i which happens with probability Ω_i, and then start a new contract which happens with probability ω_i. Hence from a single cohort $\bar{\omega}.\omega_i.\Omega_i$ will have contracts that last for exactly i periods. We then sum over the i cohorts (to include all of the contracts which are in the various stages moving towards the their final period i) to get the expression. The mean completed contract length $\bar{T}$ generated by $\boldsymbol{\omega}$ is simple to compute directly:

$$\bar{T} = \sum_{i=1}^{F} i.\alpha_i = \bar{\omega} \sum_{i=1}^{F} i^2.\omega_i.\Omega_i$$

■

9.0.2 Proposition 1(b).

Proof. Rearranging the $F-1$ equations (2) we have:

$$\frac{\alpha_1}{\bar{\omega}} = \omega_1; \frac{\alpha_2}{2\bar{\omega}} = \omega_2 (1-\omega_1) \ldots \frac{\alpha_i}{i.\bar{\omega}} = \omega_i \Omega_i; \ldots \frac{\alpha_F}{F\bar{\omega}} = \Omega_F$$

By repeated substitution starting from $i = 1$ we find that

$$\omega_i = \frac{\alpha_i}{i} \left(\bar{\omega} - \sum_{j=1}^{i-1} \frac{\alpha_j}{j} \right)^{-1} \tag{15}$$

$$\Omega_i = \frac{1}{\bar{\omega}} \left[\bar{\omega} - \sum_{j=1}^{i-1} \frac{\alpha_j}{j} \right]$$

Since we know that $\omega_F = 1$, from (15)this means that:

$$1 = \frac{\alpha_F}{F} \left(\bar{\omega} - \sum_{i=1}^{F-1} \frac{\alpha_i}{i} \right)^{-1} \Rightarrow \bar{\omega} = \sum_{i=1}^{F} \frac{\alpha_i}{i}$$

Substituting the value of $\bar{\omega}$ into (15) establishes the result. ■

9.0.3 Corollary 1(a).

Proof. The proportion of firms that have a contract that last for exactly 1 period are those that are born (age 1) and do not go on to age 2. The proportion of firms that last for exactly i periods in any one cohort (born at the same time) is given by those who attain the age i but who do not make it to $i+1$: this is $\left(\alpha_i^s - \alpha_{i+1}^s\right)$ per cohort and at any time t there are i cohorts containing contracts that will last for i periods.

Clearly, since α_j^s are monotonic, $\alpha_i \leq 1$, and

$$\begin{aligned}
\sum_{i=1}^{F} \alpha_i &= \sum_{i=1}^{F} i\left(\alpha_i^s - \alpha_{i+1}^s\right) \\
&= \left(\alpha_1^s - \alpha_2^s\right) + 2\left(\alpha_2^s - \alpha_3^s\right) - 3\left(\alpha_3^s - \alpha_4^s\right) \ldots . \\
&= \sum_{i=1}^{F} \alpha_i^s = 1
\end{aligned}$$

Hence $\boldsymbol{\alpha} \in \Delta^{F-1}$.

The relationship between the distribution of ages and lifetimes can be depicted in terms of matrix Algebra: in the case of $F = 4$:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} \alpha_1^s \\ \alpha_2^s \\ \alpha_3^s \\ \alpha_4^s \end{bmatrix}$$

Clearly, the 4×4 matrix is a mapping from $\Delta^3 \rightarrow \Delta^3$: since the matrix is of full rank, the mapping from α^s to α is $1-1$. Clearly, this holds for any F.
■

9.0.4 Corollary 1(b):

Proof. To see this, we can rewrite (3):

$$\begin{aligned}
\alpha_1 &= \alpha_1^s - \alpha_2^s \\
\frac{\alpha_2}{2} &= \left(\alpha_2^s - \alpha_3^s\right) \\
\frac{\alpha_i}{i} &= \left(\alpha_i^s - \alpha_{i+1}^s\right) \\
\frac{\alpha_F}{F} &= \alpha_F^s
\end{aligned}$$

hence summing over all possible durations $i = 1...F$ gives

$$\sum_{i=1}^{F} \frac{\alpha_i}{i} = \sum_{i=1}^{F-1} \left(\alpha_i^s - \alpha_{i+1}^s\right) + \alpha_F^s = \alpha_1^s$$

So that by repeated substitution we get:

$$\begin{aligned} \alpha_2^s &= \alpha_1^s - \alpha_1 = \sum_{i=2}^{F} \frac{\alpha_i}{i} \\ \alpha_j^s &= \sum_{i=j}^{F} \frac{\alpha_i}{i} \qquad j = 1...F \end{aligned}$$

■

9.1 Proof of Proposition 2.

Without loss of generality let $0 < \bar{\omega}_1 < \bar{\omega}_2 < .... < \bar{\omega}_n < 1$. We then have

$$\frac{\alpha_i}{i} = \sum_{k=1}^{n} \beta_k \bar{\omega}_k^2 \left(1 - \bar{\omega}_k\right)^{T-1} = \sum_{k=1}^{n} \beta_k \frac{\alpha_{ki}}{i}$$

the period i hazard rate is the ω_i is the average of the hazard rates taken over the survivors to i. Note that k and j subscripts refer to the sectoral calvo reset probabilities: ω_i the aggregate duration dependent reset, which is a weighted sum of the sectoral reset probabilities, the weights being given by ϕ_{ki}, the share of sector k survivors in all survivors:

$$\begin{aligned} \omega_i &= \sum_{k=1}^{n} \phi_k \bar{\omega}_k \\ \phi_{ki} &= \frac{\beta_k \bar{\omega}_k \left(1 - \bar{\omega}_k\right)^{i-1}}{\sum_{k=1}^{n} \beta_k \bar{\omega}_k \left(1 - \bar{\omega}_k\right)^{i-1}} \text{ share of k in survivors} \end{aligned}$$

I now divide up the proof into three steps.

Lemma 1 The share of survivors of type k at duration i is increasing (decreasing) if the hazard rate $\bar{\omega}_k$ is less than (greater than) the average hazard ω_i

Proof.

$$\begin{aligned}
\phi_{ki+1} - \phi_{ki} &= \frac{\beta_k \bar{\omega}_k (1-\bar{\omega}_k)^i}{\sum_{j=1}^n \beta_j \bar{\omega}_j (1-\bar{\omega}_j)^i} - \frac{\beta_k \bar{\omega}_k (1-\bar{\omega}_k)^{i-1}}{\sum_{j=1}^n \beta_j \bar{\omega}_j (1-\bar{\omega}_j)^{i-1}} \\
&= \frac{\beta_k \bar{\omega}_k (1-\bar{\omega}_k)^{i-1} \left[\sum_{j=1}^n \beta_j \bar{\omega}_j (1-\bar{\omega}_j)^{i-1} (\bar{\omega}_j - \bar{\omega}_k)\right]}{\left(\sum_{j=1}^n \beta_j \bar{\omega}_j (1-\bar{\omega}_j)^i\right)\left(\sum_{j=1}^n \beta_j \bar{\omega}_j (1-\bar{\omega}_j)^{i-1}\right)} \\
&= \frac{\beta_k \bar{\omega}_k (1-\bar{\omega}_k)^{i-1} (\omega_i - \bar{\omega}_k)}{\sum_{j=1}^n \beta_j \bar{\omega}_j (1-\bar{\omega}_j)^i} \\
&= \phi_{ki+1} \frac{\omega_i - \bar{\omega}_k}{1-\bar{\omega}_k}
\end{aligned}$$

which establishes Lemma 1, since $\phi_{ki+1} > 0$ and $1 - \bar{\omega}_k > 0$. ■

Lemma 2 The hazard rate decreases with duration $\omega_{i+1} < \omega_i$.

Proof.

$$\begin{aligned}
\omega_{i+1} - \omega_i &= \sum_{k=1}^n \bar{\omega}_k \left(\phi_{ki+1} - \phi_{ki}\right) \\
&= \sum_{k=1}^n \bar{\omega}_k \phi_{ki+1} \frac{\omega_i - \bar{\omega}_k}{1-\bar{\omega}_k} \\
&= \sum_{k=1}^n \frac{\bar{\omega}_k}{1-\bar{\omega}_k} \phi_{ki+1} (\omega_i - \bar{\omega}_k) < 0
\end{aligned}$$

Since the higher vales of $\bar{\omega}_k$ have higher weights, this sum is negative unless all values of $\bar{\omega}_k = \omega$, in which case the hazard is constant. ■

Lemma 3 In the limit as $i \to \infty$, $\omega_i \to \min[\bar{\omega}_k] = \bar{\omega}_1$, $\phi_{1i} \to 1$.

Proof. The share of type 1's in the survivors is

$$\begin{aligned}
\phi_{1i} &= \frac{\beta_1\bar{\omega}_1\left(1-\bar{\omega}_1\right)^{i-1}}{\sum_{k=1}^{n}\beta_k\bar{\omega}_k\left(1-\bar{\omega}_k\right)^{i-1}} \\
&= \left[1+r_2\left(\frac{1-\bar{\omega}_2}{1-\bar{\omega}_1}\right)^{i-1}...+r_n\left(\frac{1-\bar{\omega}_n}{1-\bar{\omega}_1}\right)^{i-1}\right]^{-1} \\
\text{where } r_k &= \frac{\beta_k\bar{\omega}_k}{\beta_1\bar{\omega}_1} \\
\underset{i\to\infty}{Lim}\phi_{1i}^{-1} &= \underset{i\to\infty}{Lim}\left[1+\sum_{k=2}^{n}\left[r_k\left(\frac{1-\bar{\omega}_k}{1-\bar{\omega}_1}\right)^{i-1}\right]\right]^{-1}=1 \\
\text{since } \bar{\omega}_1 &< \bar{\omega}_k \text{ for } k=2...n. \\
\text{Hence } \underset{i\to\infty}{Lim}\phi_{ki}^{-1} &= 0 \text{ for all } k>1.
\end{aligned}$$

■

9.2 Proof of Proposition 3.

Proof. Firstly we will prove (a) and (b). We do this by contradiction. Let us suppose a solution $\boldsymbol{\alpha}$ such that $\alpha_k > 0$ and $\alpha_j > 0$ and $k-j \geq 2$ We will then show that there is another feasible GTE $\boldsymbol{\alpha}'$ with $\alpha_j > 0$ and $\alpha_{j+1} > 0$ which generates a shorter average contract length.

Let us start at the proposed solution α, and in particular the two sectors k and j, whose sector shares must satisfy the two relations:

$$\begin{aligned}
\alpha_k+\alpha_j &= \rho = 1-\sum_{i=1,i\neq j,k}^{F}\alpha_i \\
\frac{\alpha_k}{k}+\frac{\alpha_j}{j} &= \eta = \bar{\omega}-\sum_{i=1,i\neq j,k}^{F}\frac{\alpha_i}{i}
\end{aligned} \tag{16}$$

ρ is the total share of the two sectors: if there are only two sectors then $\rho = 1$; if there are more than two sectors with positive shares then ρ is equal 1 minus the share of the sectors other than j and k. Likewise, η is the sum of the contribution of these two sectors to $\bar{\omega}$ less the contribution of any sectors other than j and k. Note that since $k > j$,

$$\frac{\rho}{\eta} > j \tag{17}$$

We can rewrite (16) as

$$\begin{aligned}\alpha_j &= \frac{kj}{k-j}\eta - (k-j)\rho \\ \alpha_k &= \rho(1+k-j) - \frac{kj}{k-j}\eta\end{aligned} \tag{18}$$

What we show is that we can choose $\left(\alpha'_j, \alpha_{j+1}\right)$ which satisfies the two relations above (and hence is feasible) but yields a lower average contract length. Specifically, We choose α'_{j+1}, α'_j such that

$$\begin{aligned}\alpha'_j &= j(j+1)\eta - \rho \\ \alpha'_{j+1} - \alpha_{j+1} &= 2\rho - j(j-1)\eta\end{aligned} \tag{19}$$

Define $\Delta\alpha_{j+1} = \alpha'_{j+1} - \alpha_{j+1}$. What we are doing is redistributing the total proportion ρ over durations j and $j+1$ so that the aggregate proportion of firms resetting the price is the same: $\boldsymbol{\alpha}' \in A(\bar{\omega})$,since (19) is equivalent to (18) implies

$$\begin{aligned}\Delta\alpha_{j+1} + \alpha'_j &= \rho \\ \frac{\Delta\alpha_{j+1}}{k} + \frac{\alpha'_j}{j} &= \eta\end{aligned} \tag{20}$$

Lastly, we show that $\boldsymbol{\alpha}'$ has a lower average contract length. Since we leave the proportions of other durations constant, their contribution to the average contract length is unchanged. From (18) the contribution of durations k and j under α is given by

$$\begin{aligned}T_k &= ka_k + j\alpha_j \\ &= \rho\left(k + (k-j)^2\right) - kj\eta\end{aligned}$$

Likewise the contribution of j and $j+1$ with α' is given by

$$\begin{aligned}T_{j+1} &= (j+1)\Delta\alpha_{j+1} + j\alpha'_j \\ &= \rho(j+2) - (j+1)j\eta\end{aligned}$$

Now we show that

$$T_k - T_{j+1} = \rho\left(k - (j+1) + (k-j)^2 - 1\right) - \eta\left(kj - (j+1)j\right)$$

Noting strict inequality (17) we have

$$\begin{aligned} T_k - T_{j+1} &> \eta \left[j \left(k - (j+1) + (k-j)^2 - 1 \right) - kj + (j+1)\, j \right] \\ &> \eta \left[j \left(k - j - 1 \right) \right] > 0 \end{aligned}$$

Hence

$$\bar{T}(\boldsymbol{\alpha}) - \bar{T}(\boldsymbol{\alpha}') = T_k - T_{j+1} > 0$$

the desired contradiction.

Hence, the GTE with the minimum contract length consistent with the observed $\bar{\omega}$ cannot have strictly positive sector shares which are not consecutive integers. There are at most two strictly positive sector shares.

To prove (c) for sufficiency, if $\bar{\omega}^{-1} = k \in Z_+$, then if $\alpha_k = 1 \in A(\bar{\omega})$. If $\alpha_k < 1$ any other element of $A(\bar{\omega})$ must involve strictly positive α_j and α_i with $j - i \geq 2$, which contradicts the parts (a) and (b) of the proposition already established.

For necessity, note that if $\bar{\omega}^{-1} \notin Z_+$, then no solution with only one contract length can yield the observed proportion of firms resetting prices.
■

9.3 Proof of Proposition 4.

Proof. First, note that if the proportions are given by the equations, then the rest of the proposition follows. I know show that these equations are indeed the maximising ones. Assume the contrary, that there is a distribution $\boldsymbol{\alpha}$ with $\alpha_i > 0$ where $1 < i < F$ which gives the maximum contract length. I show that this proposed optimum can be improved upon. Hence the optimum must involve only durations $\{1, F\}$ and the given equations follow automatically. So, let us take the proposed solution, with $\alpha_i > 0$. Let us redistribute the weight on sector i between $\{1, F\}$. In order to ensure that we remain in $A(\bar{\omega})$ the additional weights must satisfy

$$\begin{aligned} \Delta\alpha_i + \frac{\Delta\alpha_F}{F} &= \frac{\alpha_i}{i} \\ \Delta\alpha_i + \Delta\alpha_F &= \alpha_i \end{aligned}$$

which gives us

$$\begin{aligned} \Delta\alpha_F &= \alpha_i \frac{F(i-1)}{i(F-1)} \\ \Delta\alpha_1 &= \alpha_i \frac{F-i}{i(F-1)} \end{aligned}$$

The resulting Change in the average contract length is

$$\begin{aligned}\Delta\bar{T} &= \alpha_i \left[\frac{F(i-1)}{i(F-1)}(F-i) - \frac{F-i}{i(F-1)}(i-1)\right] \\ &= \frac{\alpha_i (i-1)(F-1)}{i(F-1)}[F-1] > 0\end{aligned}$$

The desired contradiction. Given that all contracts must be either 1 or F periods long, the rest of the proposition follows by simple algebra. ■

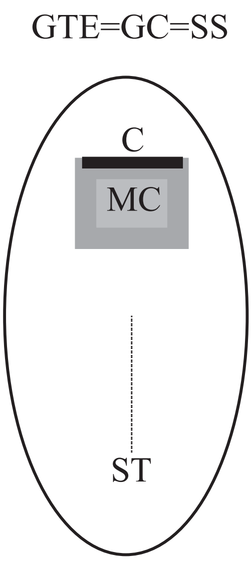


Fig.1. The Typology of contract types.

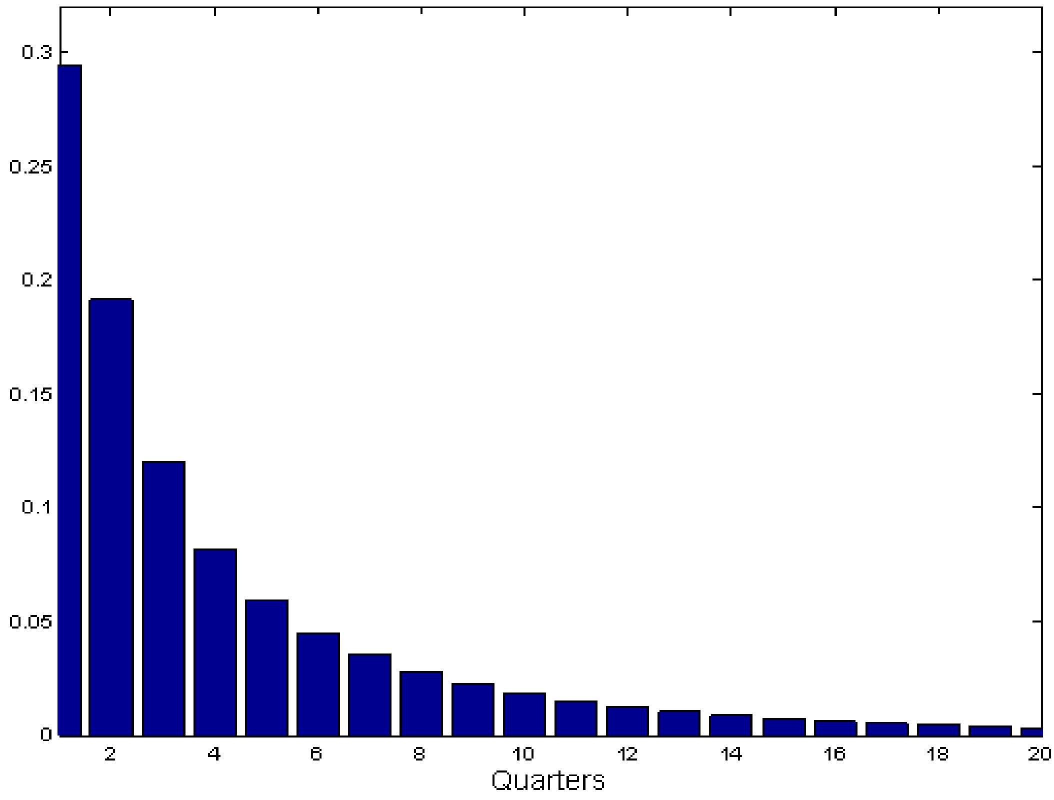


Figure 2: The BK Distribution of Contract Lenghts

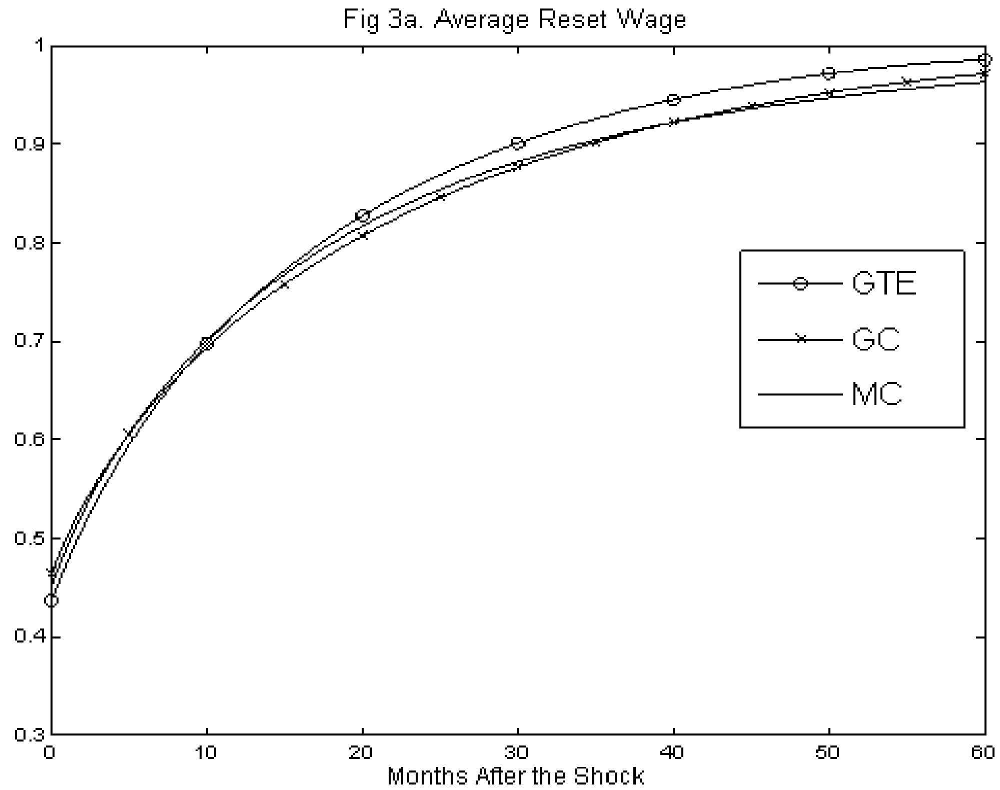


Fig 3a. Average Reset Wage
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
10
20
30
40
50
60
Months After the Shock
GTE
GC
MC

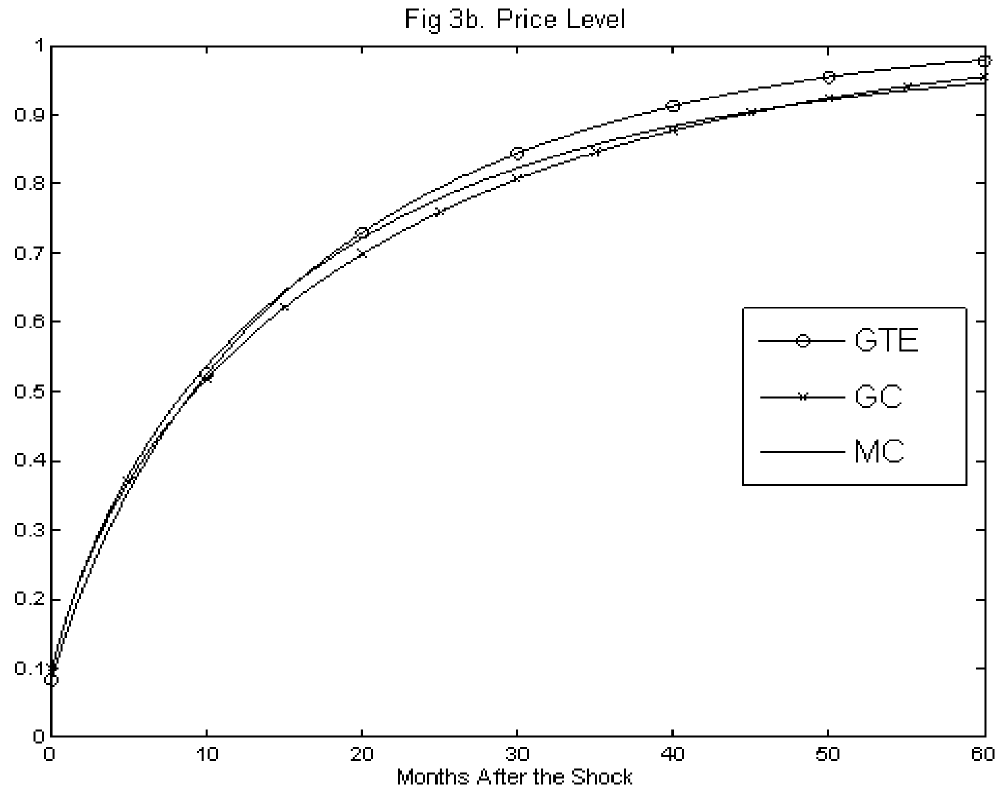


Fig 3b. Price Level
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
Months After the Shock
GTE
GC
MC

Fig 3c. Output

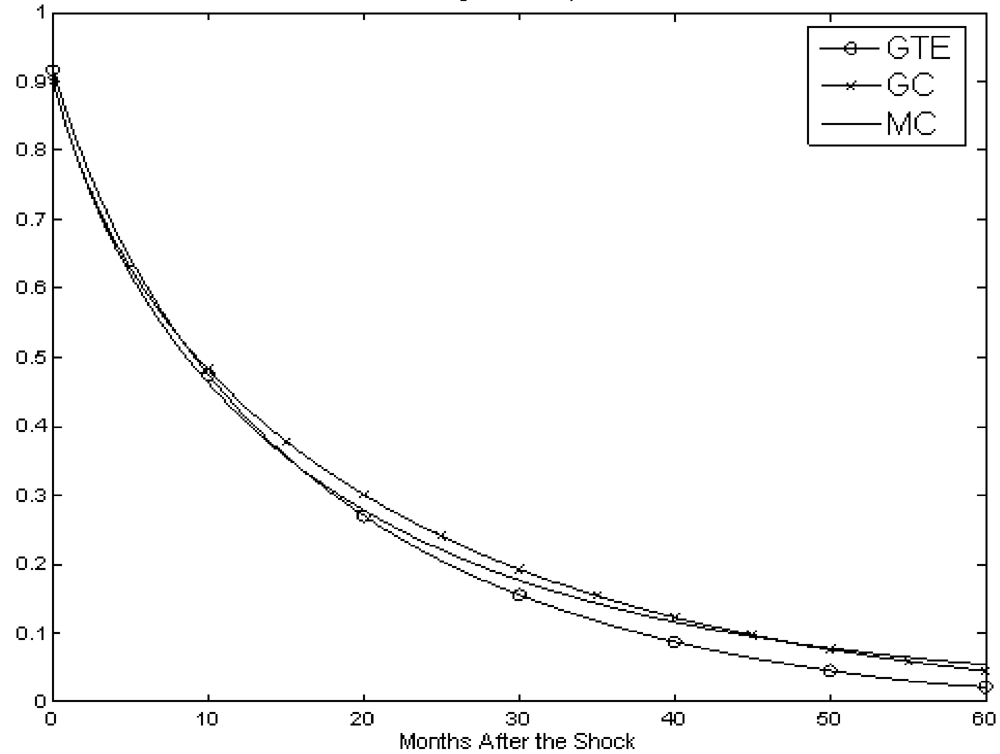
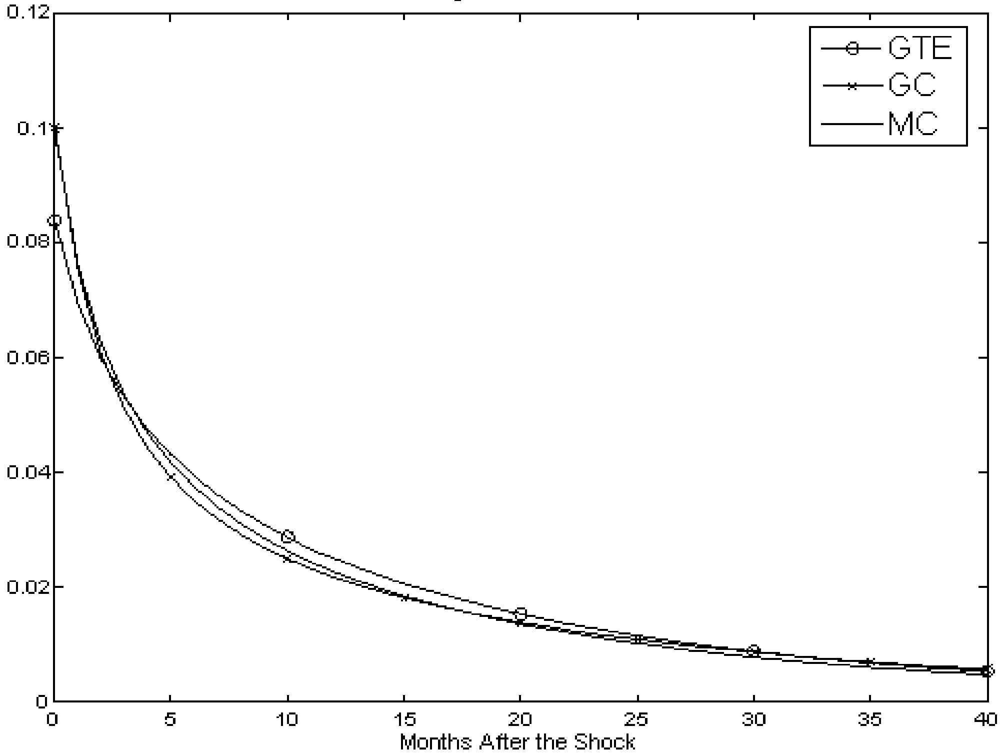


Fig 3d. Inflation



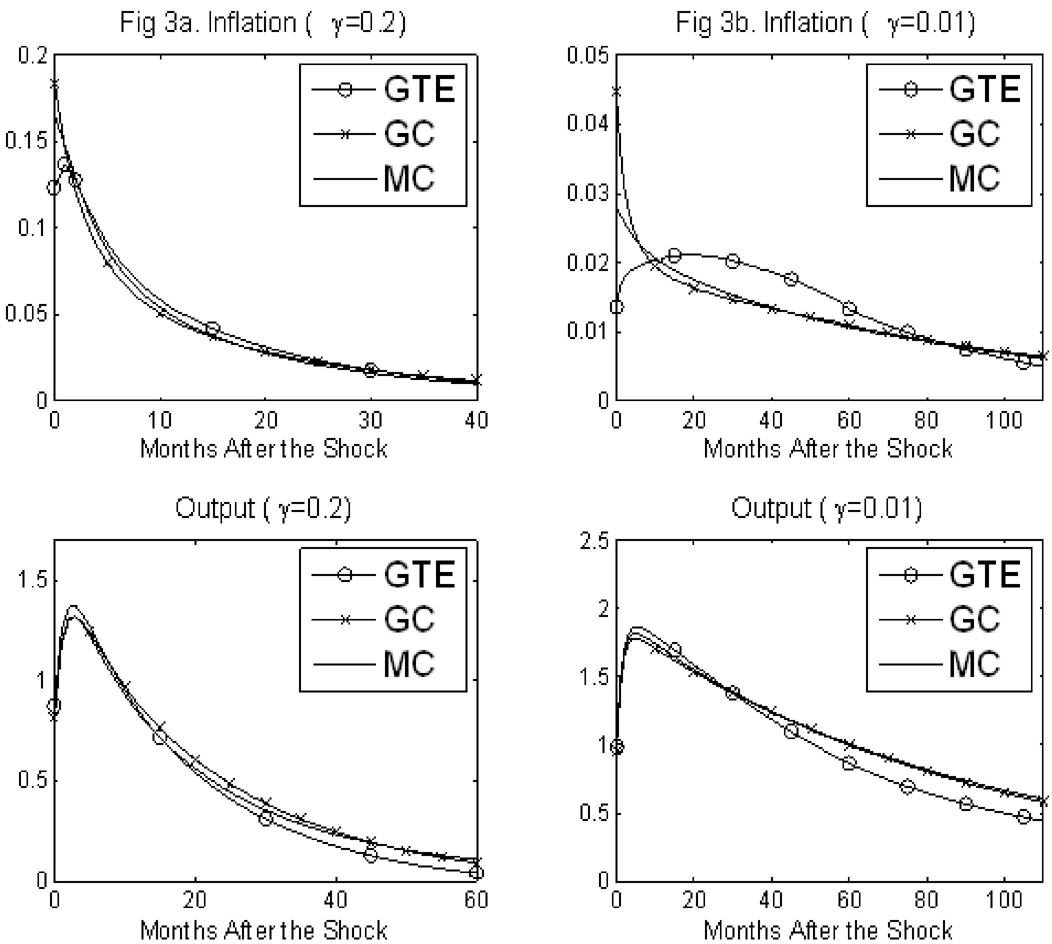


Figure 4: Serial Correlation in Monetary growth

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